CHAPTER 2

# Frequency Distributions and Graphs



## Objectives

After completing this chapter, you should be able to

1. Organize data using a frequency distribution.
2. Represent data in frequency distributions graphically using histograms, frequency polygons, and ogives.
3. Represent data using bar graphs, Pareto charts, time series graphs, and pie graphs.
4. Draw and interpret a stem and leaf plot.

## Outline

# 2-1 Organizing Data

- Data collected in original form is called **raw data**.
- A **frequency distribution** is the organization of raw data in table form, using classes and frequencies.
- Two types of frequency distributions that are most often used:

  **Categorical** and **grouped** frequency distribution

# Chapter 2
# Frequency Distributions and Graphs

Nominal- or ordinal-level data that can be placed in categories is organized in **categorical frequency distributions**.

Section 2-1

Example 2-1

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# Categorical Frequency Distribution

Twenty-five army indicates were given a blood test to determine their blood type.

Raw Data: A,B,B,AB,O O,O,B,AB,B
B,B,O,A,O A,O,O,O,AB AB,A,O,B,A

Construct a frequency distribution for the data.

# Categorical Frequency Distribution

Twenty-five army indicates were given a blood test to determine their blood type.

Raw Data: A,B,B,AB,O O,O,B,AB,B
B,B,O,A,O A,O,O,O,AB AB,A,O,B,A

| Class | Tally | Frequency | Percent |
|---|---|---|---|
| A | 𝍸 | 5 | 20 |
| B | 𝍸 II | 7 | 28 |
| O | 𝍸 IIII | 9 | 36 |
| AB | IIII | 4 | 16 |

$n = \sum f = 25$ % = f / n * 100

# Grouped Frequency Distribution

- **Grouped frequency distributions** are used when the range of the data is large.
- The smallest and largest possible data values in a class are the ***lower*** and ***upper class limits***. ***Class boundaries*** separate the classes.
- To find a class boundary, average the upper class limit of one class and the lower class limit of the next class.

Class limit (lower / upper): Between smallest and largest data value in the class.

Class boundaries: Class limit - / + .5 for whole numbers or .05 for data in tenths… rule of thumb: one additional place value as data and end in a 5) to separate the classes, no gaps in the frequency distribution.

| | | | |
|---|---|---|---|
| CL: | 10 – 20 | 10.5 – 20.5 | 10.55 - 20.5 |
| CB: | 9.5 – 20.5 | 10.45 – 20.55 | 10.545 – 20.555 |
| Rule: | -/+ .5 | -/+ .05 | -/+ .005 |

# Grouped Frequency Distribution

- The **class width** can be calculated by subtracting
  - successive lower class limits (or boundaries)
  - successive upper class limits (or boundaries)
  - upper and lower class boundaries

- The ***class midpoint*** $X_m$ can be calculated by averaging
  - upper and lower class limits (or boundaries)

$X_m$ = (lower boundary + upper boundary) / 2 or

$X_m$ = (lower limit + upper limit) / 2

Grouped frequency distribution

| Class limits | Class boundaries | Tally | Frequency | Cumulative frequency |
|---|---|---|---|---|
| 24–30 | 23.5–30.5 | /// | 3 | 3 |
| 31–37 | 30.5–37.5 | / | 1 | 4 |
| 38–44 | 37.5–44.5 | //// | 5 | 9 |
| 45–51 | 44.5–51.5 | //// //// | 9 | 18 |
| 52–58 | 51.5–58.5 | //// / | 6 | 24 |
| 59–65 | 58.5–65.5 | / | 1 | 25 |
| | | | 25 | |

7 7 7 7 7

Open end: 59+

Gap: if frequency of 31-37=0

# Rules for Classes in Grouped Frequency Distributions

1. There should be 5-20 classes.
2. The class width should be an odd number.
3. The classes must be mutually exclusive.
4. The classes must be continuous.
5. The classes must be exhaustive.
6. The classes must be equal in width (except in open-ended distributions).

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-1

Example 2-2

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# Constructing a Grouped Frequency Distribution

The following data represent the record high temperatures for each of the 50 states. Construct a grouped frequency distribution for the data using 7 classes.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 112 | 100 | 127 | 120 | 134 | 118 | 105 | 110 | 109 | 112 |
| 110 | 118 | 117 | 116 | 118 | 122 | 114 | 114 | 105 | 109 |
| 107 | 112 | 114 | 115 | 118 | 117 | 118 | 122 | 106 | 110 |
| 116 | 108 | 110 | 121 | 113 | 120 | 119 | 111 | 104 | 111 |
| 120 | 113 | 120 | 117 | 105 | 110 | 118 | 112 | 114 | 114 |

# Constructing a Grouped Frequency Distribution

STEP 1 Determine the classes.

Find the class width by dividing the range by the number of classes 7.

$$\textit{Range} = \textit{High} - \textit{Low}$$

$$= 134 - 100 = 34$$

$$\textit{Width} = \textit{Range} \,/\, \text{number of classes}$$

$$= 34 / 7 = 4.9 = 5$$

Rounding Rule: Always round up if a remainder.

# Constructing a Grouped Frequency Distribution

- For convenience sake, we will choose the lowest data value, 100, for the first lower class limit.
- The subsequent lower class limits are found by adding the width to the previous lower class limits.

| Class Limits |
| --- |
| 100 - 104 |
| 105 - 109 |
| 110 - 114 |
| 115 - 119 |
| 120 - 124 |
| 125 - 129 |
| 130 - 134 |

- The first upper class limit is one less than the next lower class limit.
- The subsequent upper class limits are found by adding the width to the previous upper class limits.

# Constructing a Grouped Frequency Distribution

- The class boundary is midway between an upper class limit and a subsequent lower class limit. 104,104.5,105

| Class Limits | Class Boundaries | Frequency | Cumulative Frequency |
|---|---|---|---|
| 100 - 104 | 99.5 - 104.5 | | |
| 105 - 109 | 104.5 - 109.5 | | |
| 110 - 114 | 109.5 - 114.5 | | |
| 115 - 119 | 114.5 - 119.5 | | |
| 120 - 124 | 119.5 - 124.5 | | |
| 125 - 129 | 124.5 - 129.5 | | |
| 130 - 134 | 129.5 - 134.5 | | |

# Constructing a Grouped Frequency Distribution

STEP 2 Tally the data.

STEP 3 Find the frequencies.

| Class Limits | Class Boundaries | Frequency | Cumulative Frequency |
|---|---|---|---|
| 100 - 104 | 99.5 - 104.5 | 2 | |
| 105 - 109 | 104.5 - 109.5 | 8 | |
| 110 - 114 | 109.5 - 114.5 | 18 | |
| 115 - 119 | 114.5 - 119.5 | 13 | |
| 120 - 124 | 119.5 - 124.5 | 7 | |
| 125 - 129 | 124.5 - 129.5 | 1 | |
| 130 - 134 | 129.5 - 134.5 | 1 | |

# Constructing a Grouped Frequency Distribution

STEP 4 Find the cumulative frequencies by keeping a running total of the frequencies.

| Class Limits | Class Boundaries | Frequency | Cumulative Frequency |
|---|---|---|---|
| 100 - 104 | 99.5 - 104.5 | 2 | 2 |
| 105 - 109 | 104.5 - 109.5 | 8 | 10 |
| 110 - 114 | 109.5 - 114.5 | 18 | 28 |
| 115 - 119 | 114.5 - 119.5 | 13 | 41 |
| 120 - 124 | 119.5 - 124.5 | 7 | 48 |
| 125 - 129 | 124.5 - 129.5 | 1 | 49 |
| 130 - 134 | 129.5 - 134.5 | 1 | 50 |

# 2-2 Histograms, Frequency Polygons, and Ogives

## 3 Most Common Graphs in Research

1. **Histogram**
2. **Frequency Polygon**
3. **Cumulative Frequency Polygon (Ogive)**

# 2-2 Histograms, Frequency Polygons, and Ogives

The ***histogram*** is a graph that displays the data by using contiguous vertical bars (unless the frequency of the class is 0) of various heights to represent the frequencies of the classes.

The class boundaries are represented on the horizontal axis.

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-2

Example 2-4

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# Histograms

Construct a histogram to represent the data for the record high temperatures for each of the 50 states (see Example 2–2 for the data).

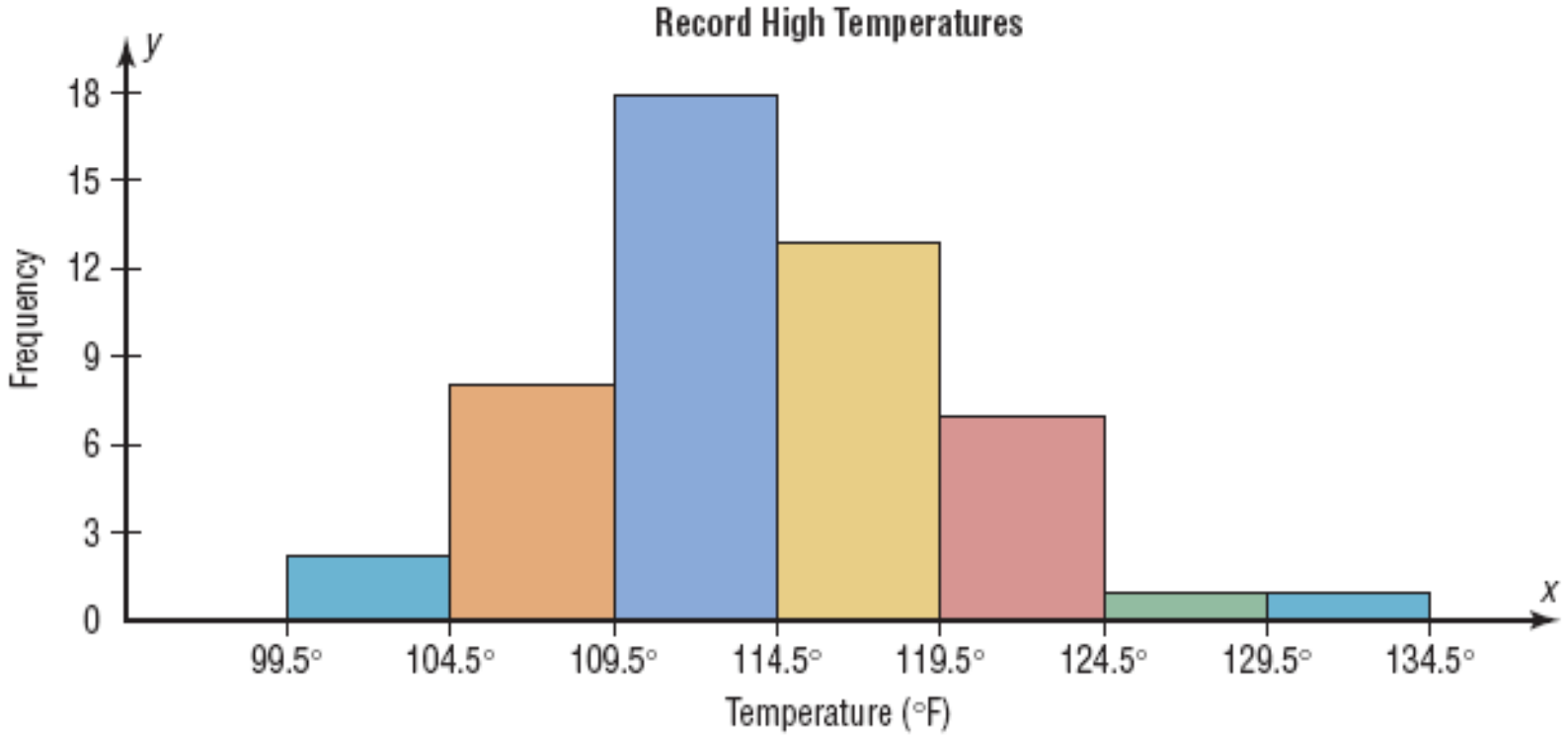
# Histograms

Histograms use class boundaries and frequencies of the classes.

| Class Limits | Class Boundaries | Frequency |
|---|---|---|
| 100 - 104 | 99.5 - 104.5 | 2 |
| 105 - 109 | 104.5 - 109.5 | 8 |
| 110 - 114 | 109.5 - 114.5 | 18 |
| 115 - 119 | 114.5 - 119.5 | 13 |
| 120 - 124 | 119.5 - 124.5 | 7 |
| 125 - 129 | 124.5 - 129.5 | 1 |
| 130 - 134 | 129.5 - 134.5 | 1 |

# Histograms

Histograms use class boundaries and frequencies of the classes.



# 2.2 Histograms, Frequency Polygons, and Ogives

- The ***frequency polygon*** is a graph that displays the data by using lines that connect points plotted for the frequencies at the class midpoints. The frequencies are represented by the heights of the points.
- The class midpoints are represented on the horizontal axis.

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-2

Example 2-5

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# Frequency Polygons

Construct a frequency polygon to represent the data for the record high temperatures for each of the 50 states (see Example 2–2 for the data).

# Frequency Polygons

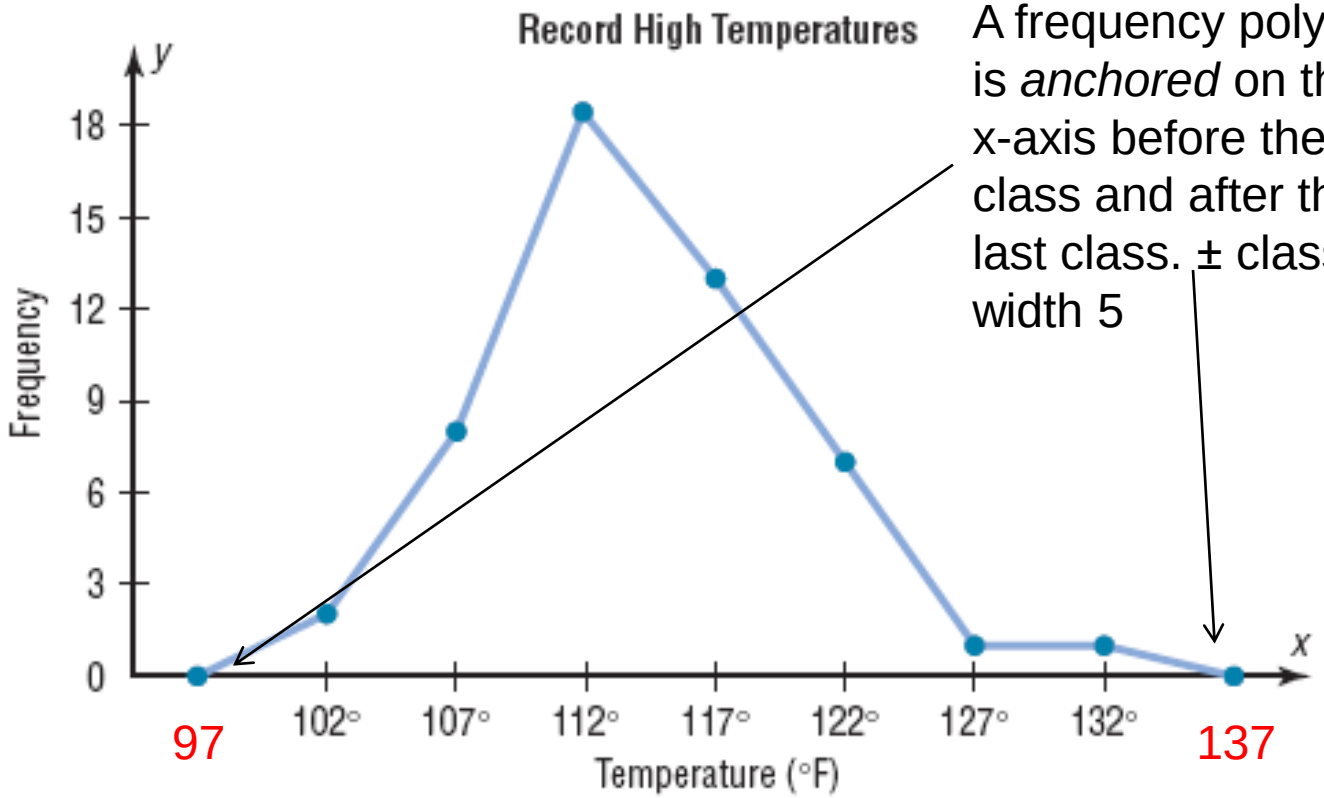
Frequency polygons use class midpoints and frequencies of the classes.

| Class Limits | Class Midpoints | Frequency |
|---|---|---|
| 100 - 104 | 102 | 2 |
| 105 - 109 | 107 | 8 |
| 110 - 114 | 112 | 18 |
| 115 - 119 | 117 | 13 |
| 120 - 124 | 122 | 7 |
| 125 - 129 | 127 | 1 |
| 130 - 134 | 132 | 1 |

***Xm***:
by averaging upper and lower class limits (or boundaries)

# Frequency Polygons

Frequency polygons use class midpoints and frequencies of the classes.




A frequency polygon is *anchored* on the x-axis before the first class and after the last class. ± class width 5

# 2.2 Histograms, Frequency Polygons, and Ogives

- The ***ogive*** is a graph that represents the cumulative frequencies for the classes in a frequency distribution.

- The upper class boundaries are represented on the horizontal axis.

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-2

Example 2-6

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# Ogives

Construct an ogive to represent the data for the record high temperatures for each of the 50 states (see Example 2–2 for the data).

# Ogives

Ogives use upper class boundaries and cumulative frequencies of the classes.

| Class Limits | Class Boundaries | Frequency | Cumulative Frequency |
|---|---|---|---|
| 100 - 104 | 99.5 - 104.5 | 2 | 2 |
| 105 - 109 | 104.5 - 109.5 | 8 | 10 |
| 110 - 114 | 109.5 - 114.5 | 18 | 28 |
| 115 - 119 | 114.5 - 119.5 | 13 | 41 |
| 120 - 124 | 119.5 - 124.5 | 7 | 48 |
| 125 - 129 | 124.5 - 129.5 | 1 | 49 |
| 130 - 134 | 129.5 - 134.5 | 1 | 50 |

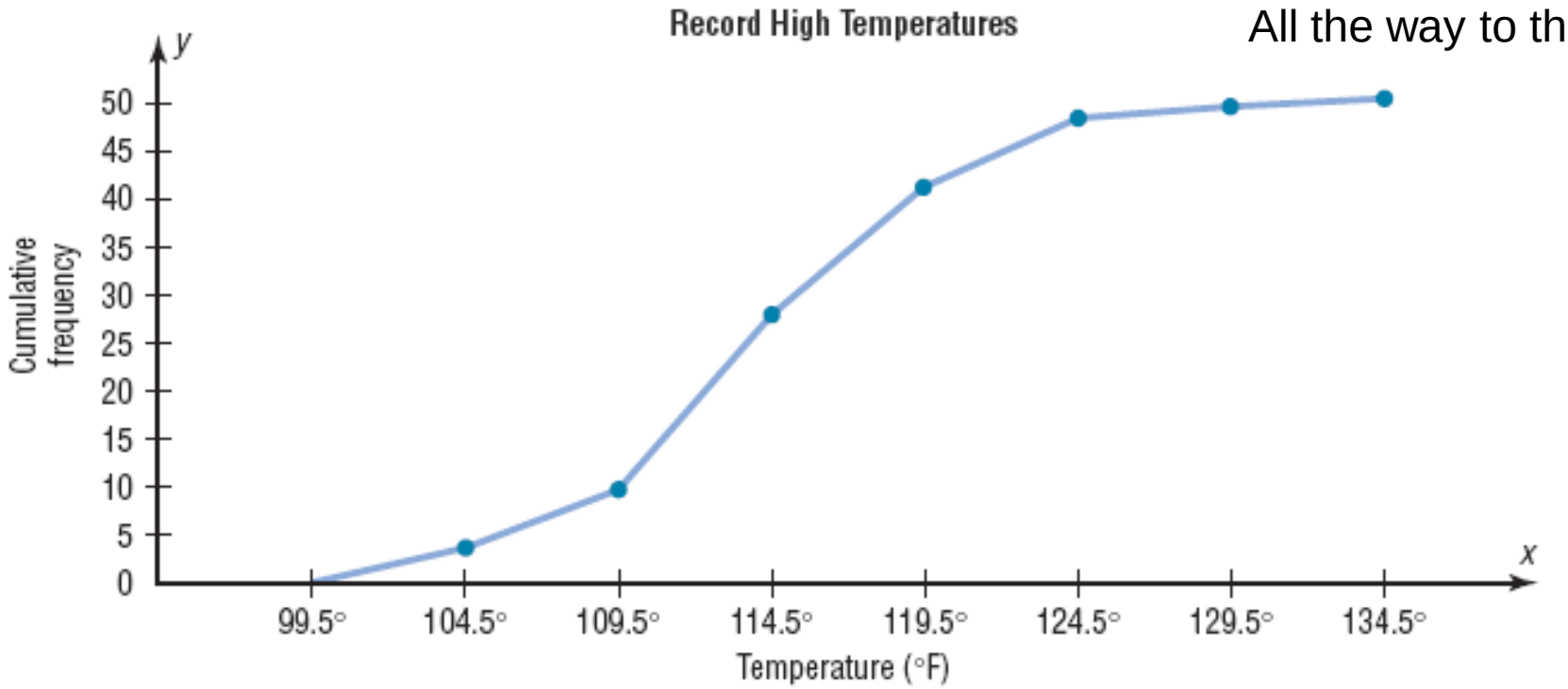
# Ogives

Ogives use upper class boundaries and cumulative frequencies of the classes.

| Class Boundaries | Cumulative Frequency |
|---|---|
| Less than 104.5 | 2 |
| Less than 109.5 | 10 |
| Less than 114.5 | 28 |
| Less than 119.5 | 41 |
| Less than 124.5 | 48 |
| Less than 129.5 | 49 |
| Less than 134.5 | 50 |

# Ogives

Ogives use upper class boundaries and cumulative frequencies of the classes.




All the way to the hill

# Procedure Table

Constructing Statistical Graphs

1: Draw and label the *x* and *y* axes.

2: Choose a suitable scale for the frequencies or cumulative frequencies, and label it on the *y* axis.

3: Represent the class boundaries for the histogram or ogive, or the midpoint for the frequency polygon, on the *x* axis.

4: Plot the points and then draw the bars or lines.

# 2.2 Histograms, Frequency Polygons, and Ogives

If proportions are used instead of frequencies, the graphs are called ***relative frequency graphs***.

Relative frequency graphs are used when the proportion of data values that fall into a given class is more important than the actual number of data values that fall into that class.

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-2

Example 2-7

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Construct a histogram, frequency polygon, and ogive using relative frequencies for the distribution (shown here) of the miles that 20 randomly selected runners ran during a given week.

| Class Boundaries | Frequency |
|---|---|
| 5.5 - 10.5 | 1 |
| 10.5 - 15.5 | 2 |
| 15.5 - 20.5 | 3 |
| 20.5 - 25.5 | 5 |
| 25.5 - 30.5 | 4 |
| 30.5 - 35.5 | 3 |
| 35.5 - 40.5 | 2 |

# Histograms

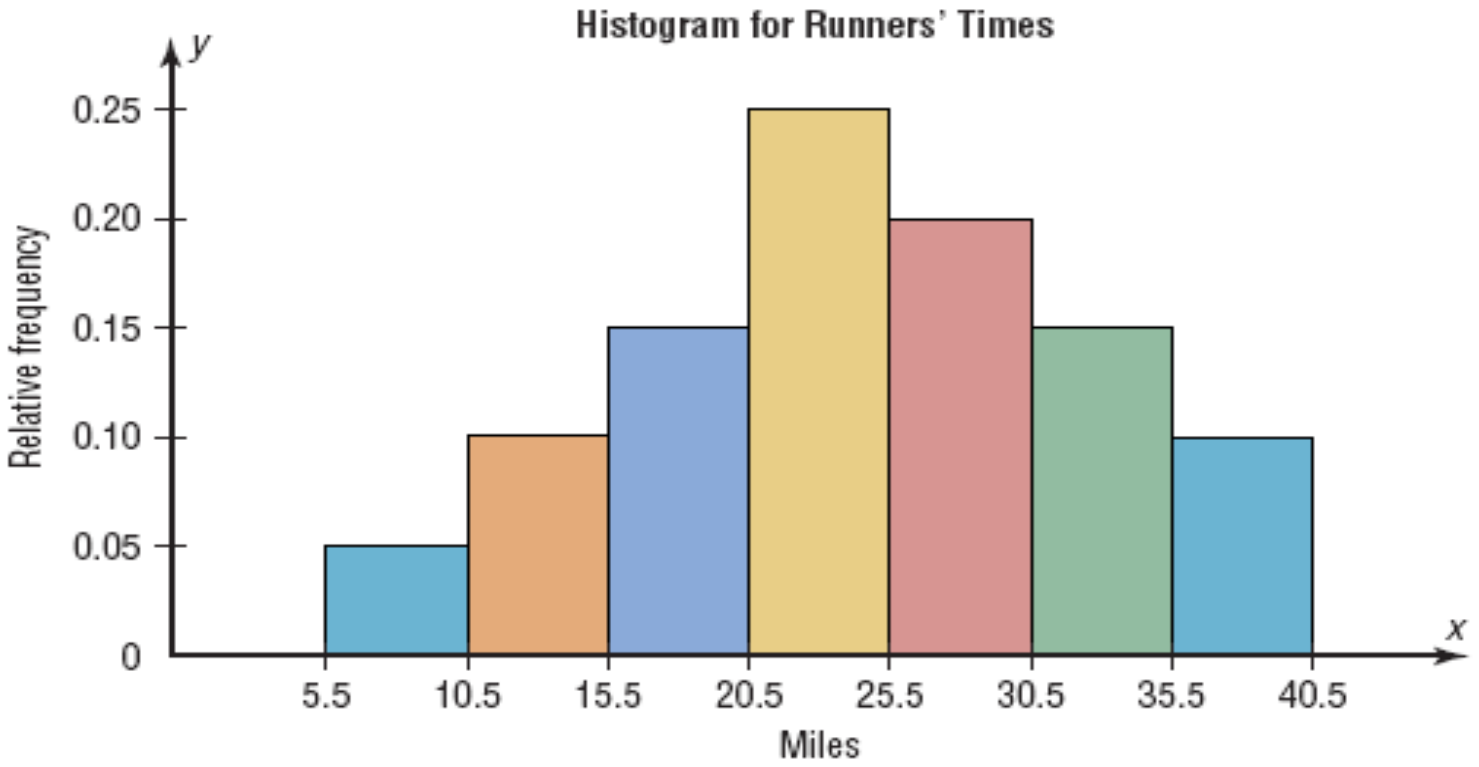
The following is a frequency distribution of miles run per week by 20 selected runners.

| Class Boundaries | Frequency | Relative Frequency |
|---|---|---|
| 5.5 - 10.5 | 1 | 1/20 = 0.05 |
| 10.5 - 15.5 | 2 | 2/20 = 0.10 |
| 15.5 - 20.5 | 3 | 3/20 = 0.15 |
| 20.5 - 25.5 | 5 | 5/20 = 0.25 |
| 25.5 - 30.5 | 4 | 4/20 = 0.20 |
| 30.5 - 35.5 | 3 | 3/20 = 0.15 |
| 35.5 - 40.5 | 2 | 2/20 = 0.10 |
| | $\Sigma f = 20$ | $\Sigma rf = 1.00$ |

Divide each frequency by the total frequency to get the relative frequency (rf = f / n) .

# Histograms

Use the class boundaries and the relative frequencies of the classes.



# Frequency Polygons

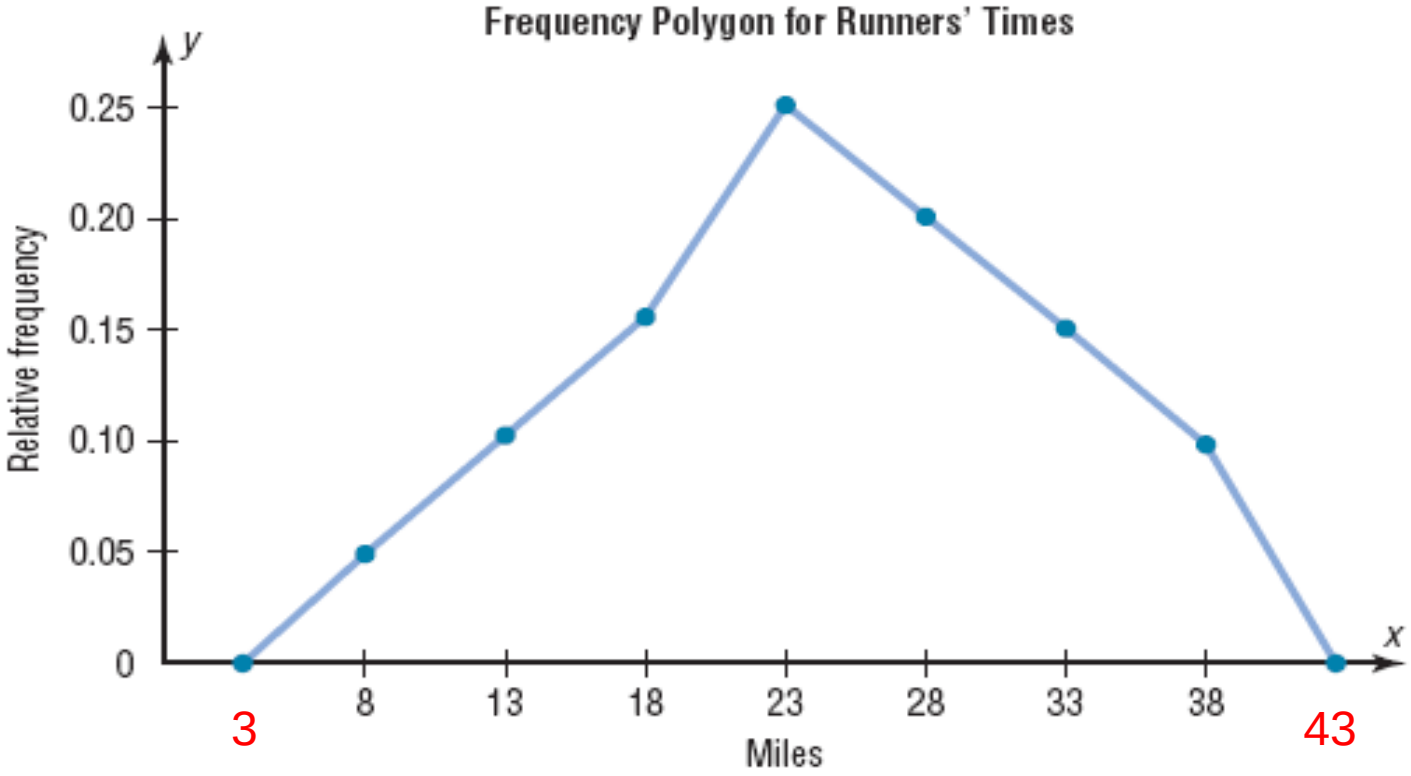
The following is a frequency distribution of miles run per week by 20 selected runners.

| Class Boundaries | Class Midpoints | Relative Frequency |
|---|---|---|
| 5.5 - 10.5 | 8 | 0.05 |
| 10.5 - 15.5 | 13 | 0.10 |
| 15.5 - 20.5 | 18 | 0.15 |
| 20.5 - 25.5 | 23 | 0.25 |
| 25.5 - 30.5 | 28 | 0.20 |
| 30.5 - 35.5 | 33 | 0.15 |
| 35.5 - 40.5 | 38 | 0.10 |

***Xm***:
by averaging upper and lower class limits (or boundaries)

# Frequency Polygons

Use the class midpoints and the relative frequencies of the classes.



# Ogives

The following is a frequency distribution of miles run per week by 20 selected runners.

| Class Boundaries | Frequency | Cumulative Frequency | Cum. Rel. Frequency |
|---|---|---|---|
| 5.5 - 10.5 | 1 | 1 | 1/20 = 0.05 |
| 10.5 - 15.5 | 2 | 3 | 3/20 = 0.15 |
| 15.5 - 20.5 | 3 | 6 | 6/20 = 0.30 |
| 20.5 - 25.5 | 5 | 11 | 11/20 = 0.55 |
| 25.5 - 30.5 | 4 | 15 | 15/20 = 0.75 |
| 30.5 - 35.5 | 3 | 18 | 18/20 = 0.90 |
| 35.5 - 40.5 | 2 | 20 | 20/20 = 1.00 |
| | $\Sigma f = 20$ | | |

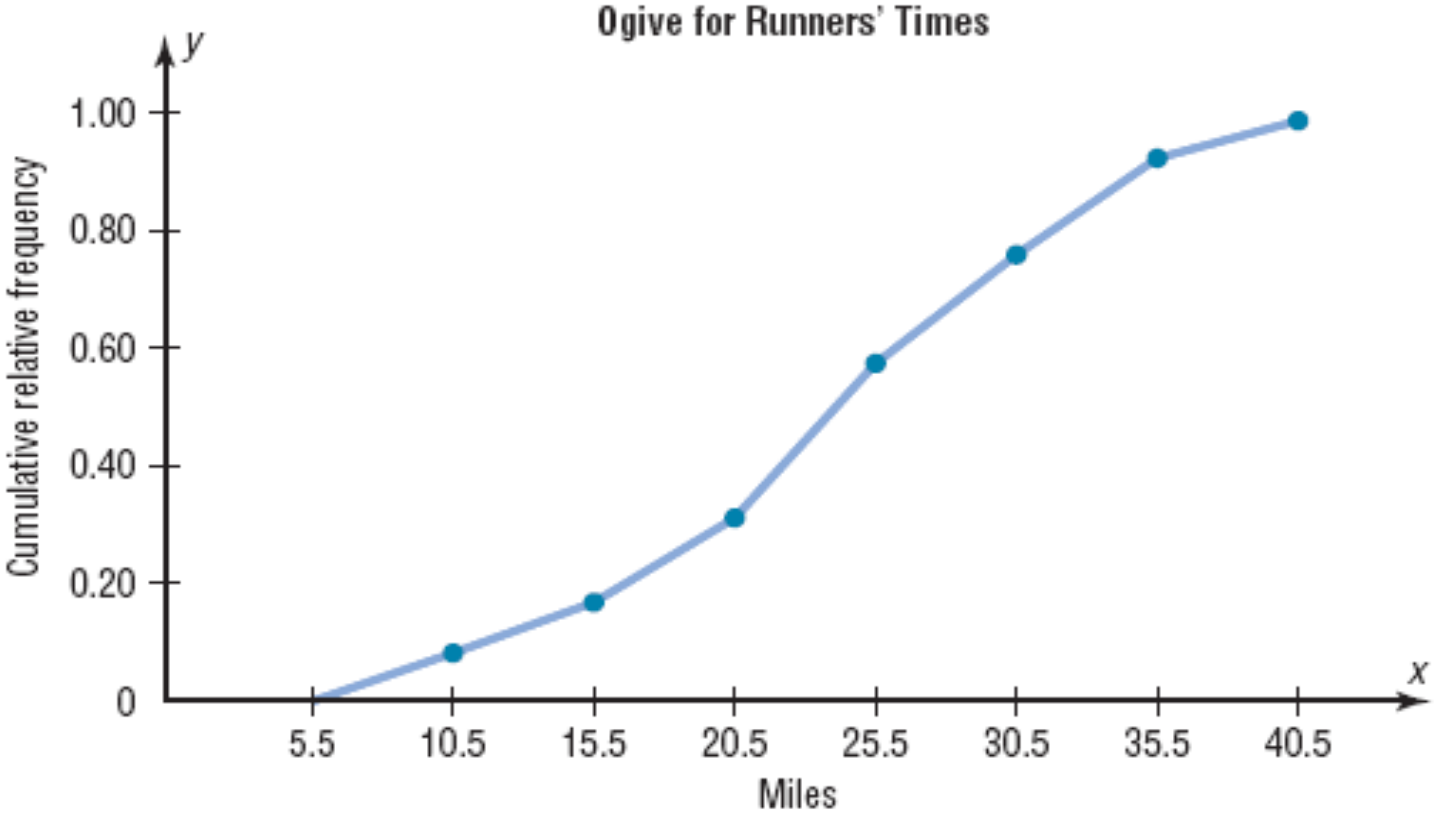
# Ogives

Ogives use upper class boundaries and cumulative frequencies of the classes.

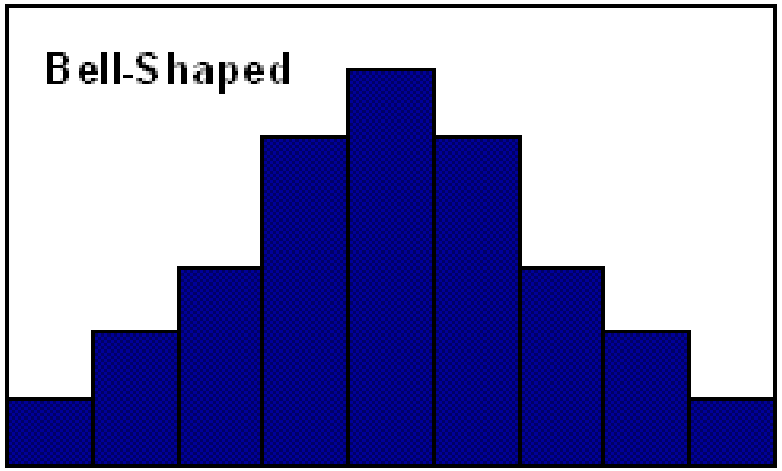
| Class Boundaries | Cum. Rel. Frequency |
|---|---|
| Less than 10.5 | 0.05 |
| Less than 15.5 | 0.15 |
| Less than 20.5 | 0.30 |
| Less than 25.5 | 0.55 |
| Less than 30.5 | 0.75 |
| Less than 35.5 | 0.90 |
| Less than 40.5 | 1.00 |

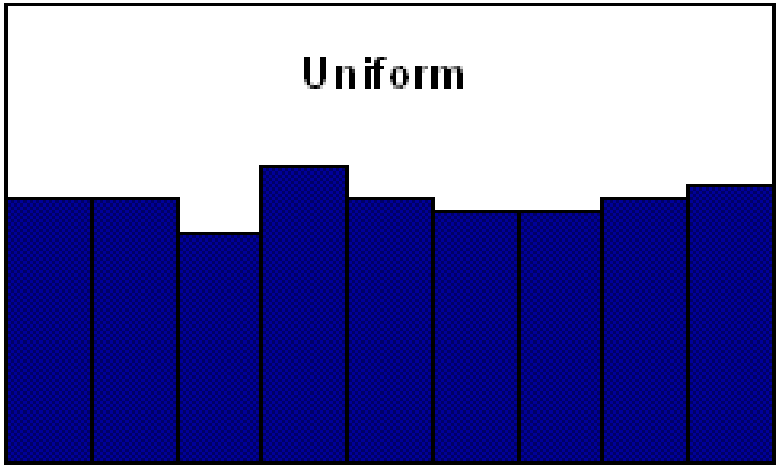
# Ogives

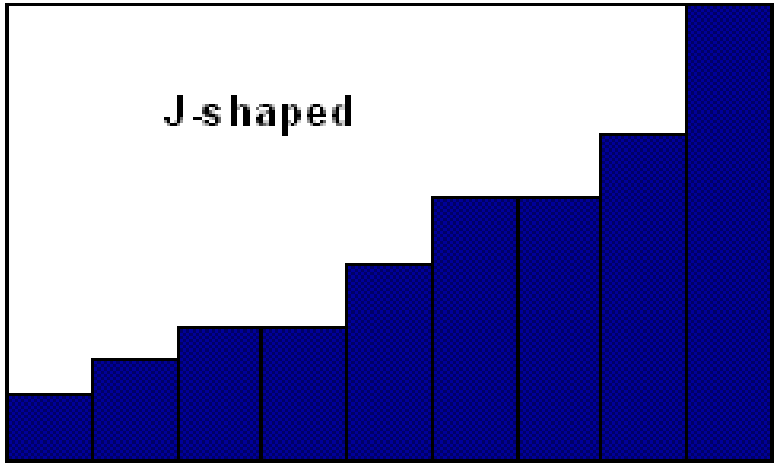
Use the upper class boundaries and the cumulative relative frequencies.

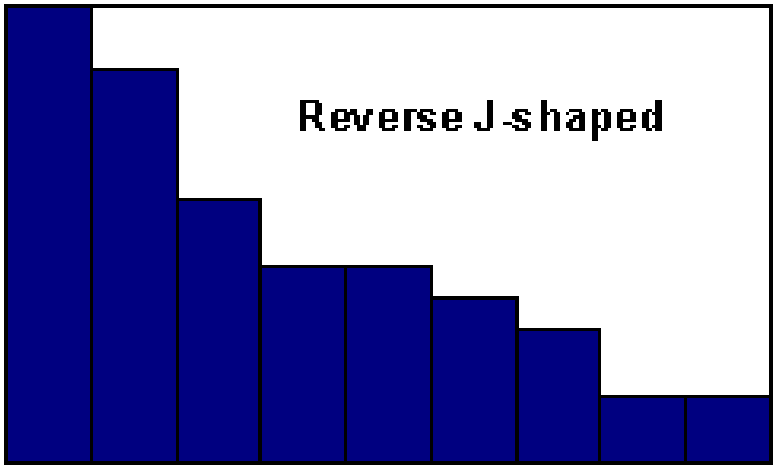


# Shapes of Distributions

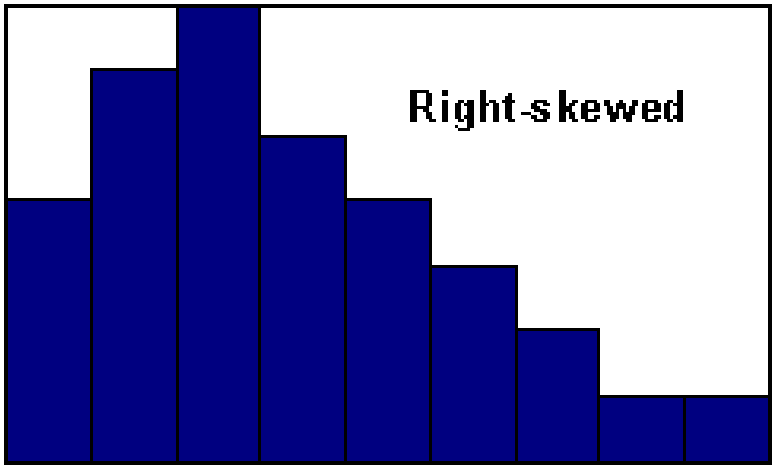


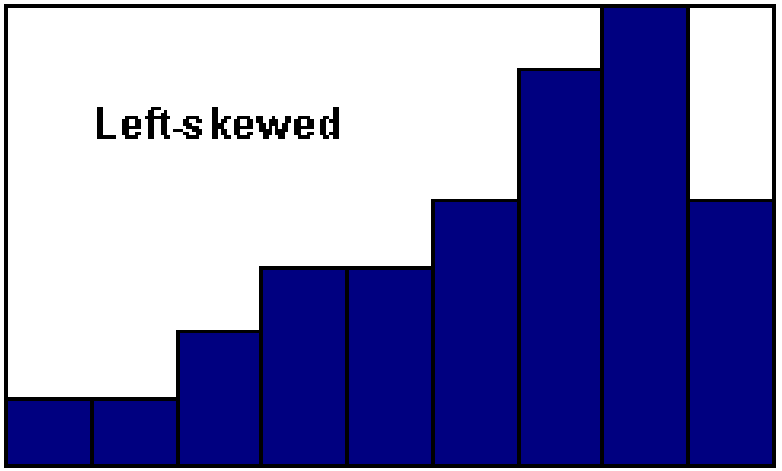



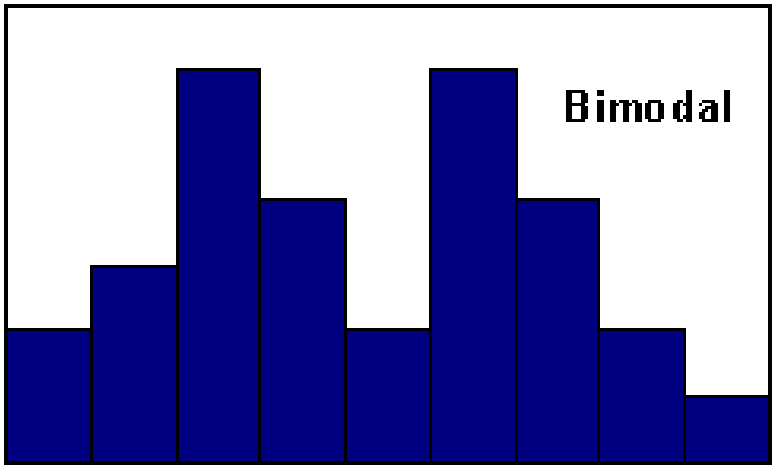



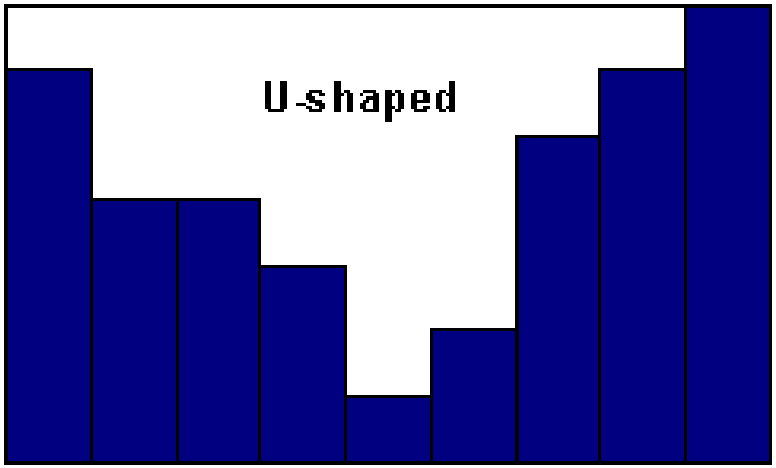



# Shapes of Distributions



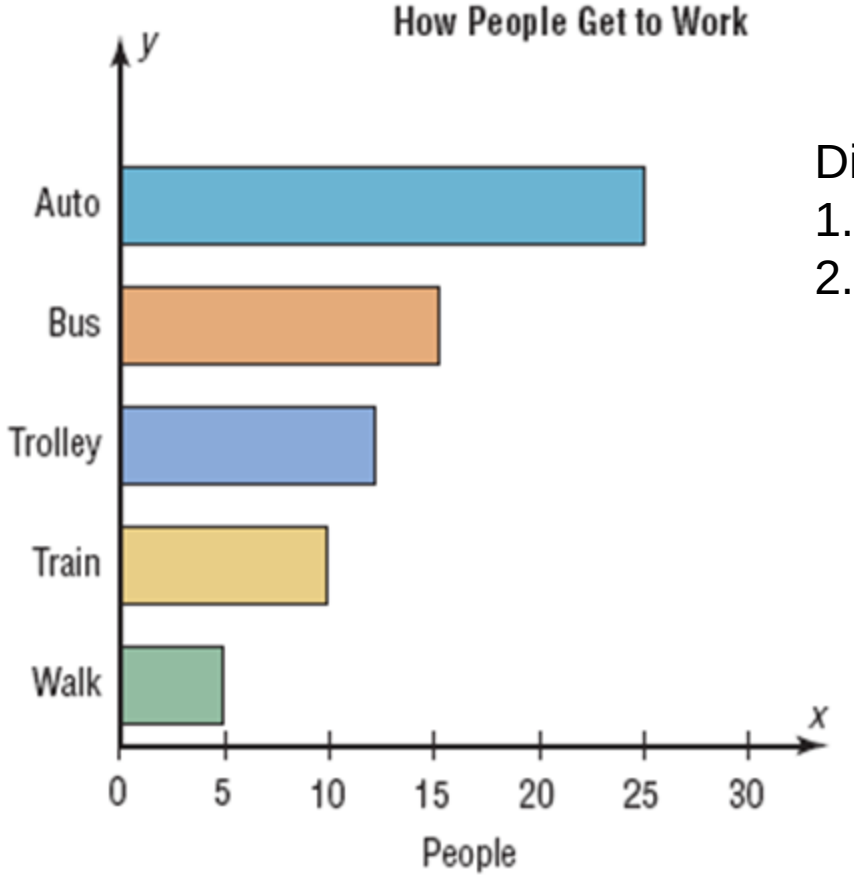



# 2.3 Other Types of Graphs

## Bar Graphs:

A **bar graph** represents the data by using vertical or horizontal bars whose heights or lengths represent the frequencies of the data.



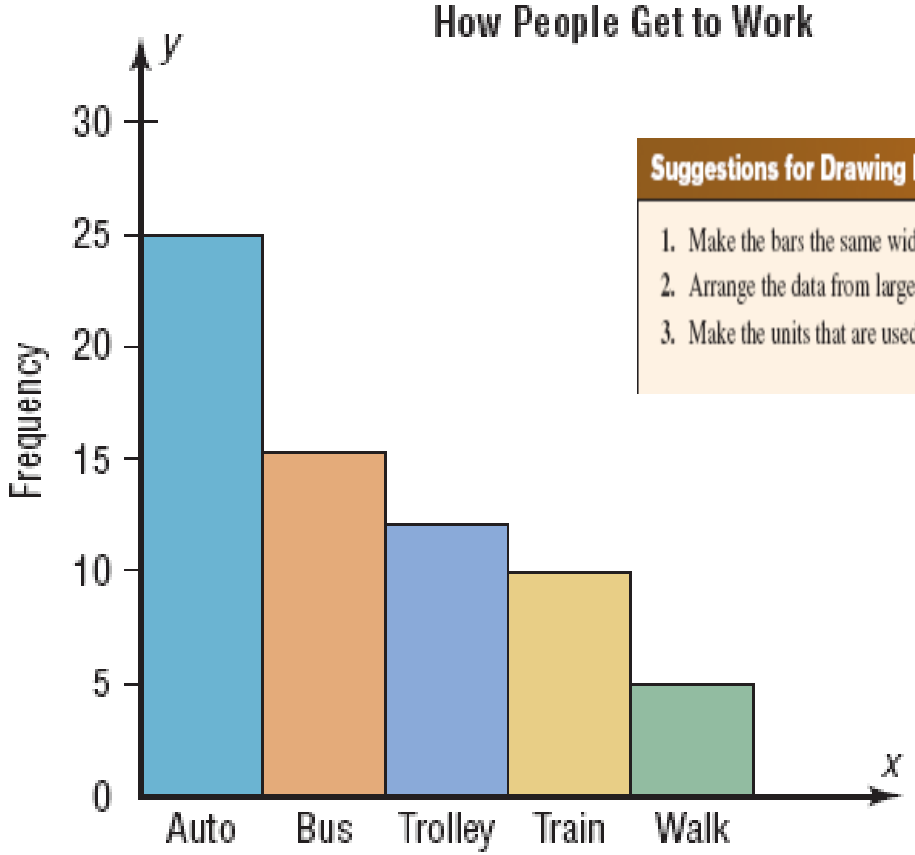

Differences than histogram?

1. For categorical data.
2. Not contiguous bar.

# 2.3 Other Types of Graphs

## Pareto Charts:

A **Pareto chart** is used to represent a frequency distribution for a categorical variable, and the frequencies are displayed by the heights of vertical bars, which are arranged in order from highest to lowest.




**Suggestions for Drawing Pareto Charts**

1. Make the bars the same width.
2. Arrange the data from largest to smallest according to frequency.
3. Make the units that are used for the frequency equal in size.

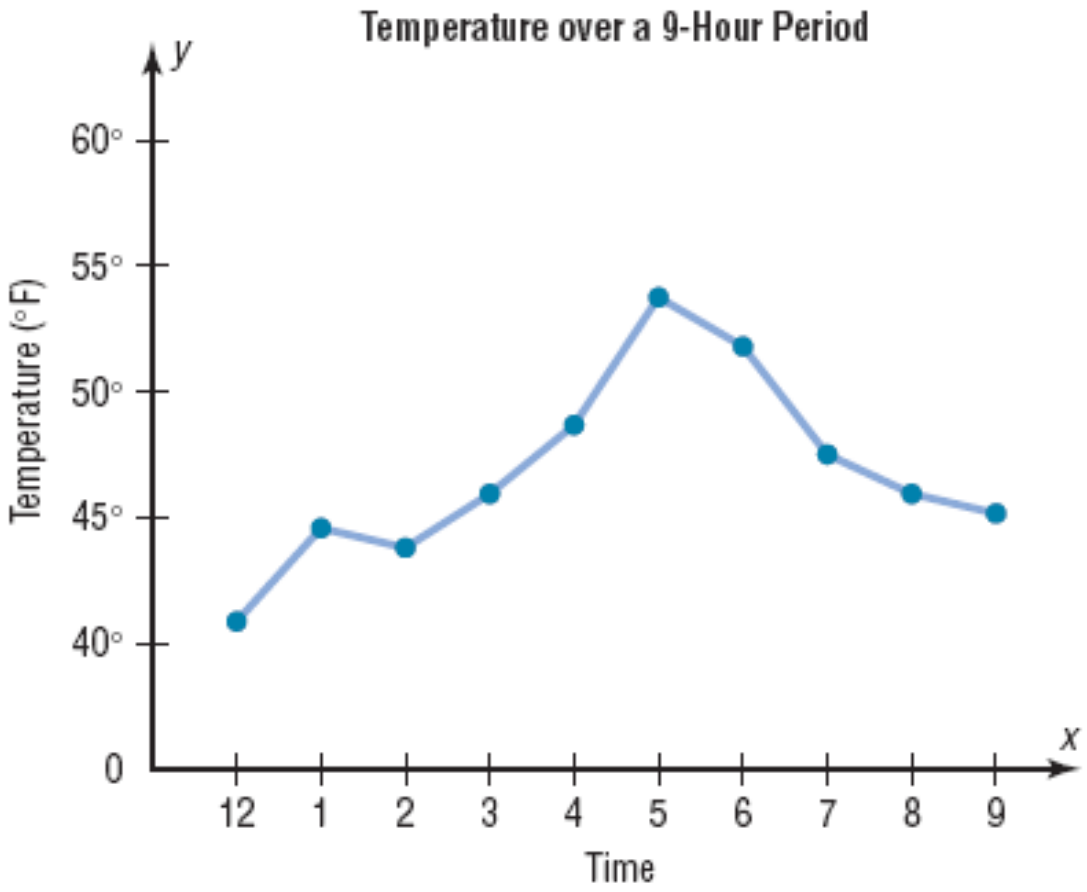
Differences than bar chart?

1. Arrange the data from largest to smallest according to frequency.
2. Contiguous bar.

# 2.3 Other Types of Graphs

## Time Series Graphs:

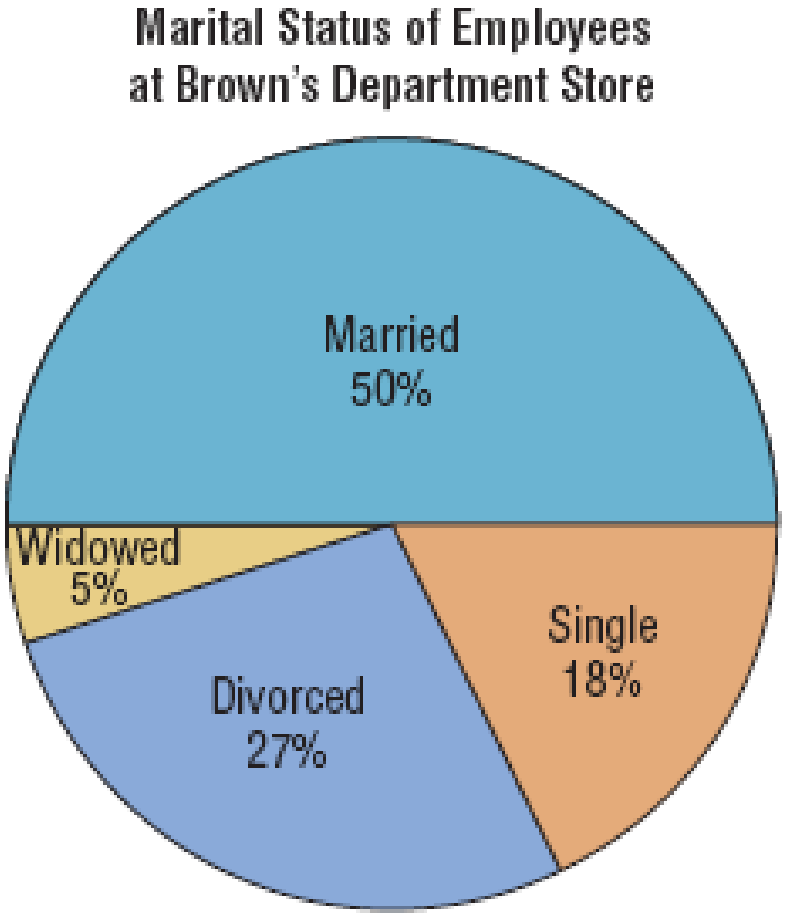
**A time series graph** represents data that occur over a specific period of time.



# 2.3 Other Types of Graphs

## Pie Graphs:

A **pie graph** is a circle that is divided into sections or wedges according to the percentage of frequencies in each category of the distribution.




| Percent* | Degree** |
|---|---|
| 50% * 360 = | 180 |
| 18% * 360 = | 64.8 |
| 27% * 360 = | 97.2 |
| 5% * 360 = | 18 |

* Percent = (f / n) * 100%

** Degree = % * 360

# 2.3 Other Types of Graphs
# Stem and Leaf Plots

A ***stem and leaf plots*** is a data plot that uses part of a data value as the stem and part of the data value as the leaf to form groups or classes.

It has the advantage over grouped frequency distribution of retaining the actual data while showing them in graphic form.

# Chapter 2
# Frequency Distributions and Graphs

## Section 2-3

Example 2-13

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At an outpatient testing center, the number of cardiograms performed each day for 20 days is shown. Construct a stem and leaf plot for the data.

| | | | | |
|---|---|---|---|---|
| 25 | 31 | 20 | 32 | 13 |
| 14 | 43 | 2 | 57 | 23 |
| 36 | 32 | 33 | 32 | 44 |
| 32 | 52 | 44 | 51 | 45 |

Raw data:

| | | | | |
|---|---|---|---|---|
| 25 | 31 | 20 | 32 | 13 |
| 14 | 43 | 2 | 57 | 23 |
| 36 | 32 | 33 | 32 | 44 |
| 32 | 52 | 44 | 51 | 45 |

Data array:

02
13 14
20 23 25
31 32 32 32 32 33 36
43 44 44 45
51 52 57

## Ordered Stem and leaf Plot

Leading digit (stem) Trailing digit (leaf)

| | |
|---|---|
| 0 | 2 |
| 1 | 3 4 |
| 2 | 0 3 5 |
| 3 | 1 2 2 2 2 3 6 |
| 4 | 3 4 4 5 |
| 5 | 1 2 7 |