CHAPTER

4

Probability and Counting Rules

Objectives

After completing this chapter, you should be able to

1. Determine sample spaces and find the probability of an event, using classical probability or empirical probability.
2. Find the probability of compound events, using the addition rules.
3. Find the probability of compound events, using the multiplication rules.
4. Find the conditional probability of an event.
5. Find the total number of outcomes in a sequence of events, using the fundamental counting rule.
6. Find the number of ways that *r* objects can be selected from *n* objects, using the permutation rule.
7. Find the number of ways that *r* objects can be selected from *n* objects without regard to order, using the combination rule.
8. Find the probability of an event, using the counting rules.

Outline

Probability

- **Probability (P)** can be defined as the chance of an event occurring. It can be used to quantify what the “odds” are that a specific event will occur. Some examples of how probability is used everyday would be weather forecasting, “75% chance of snow” or for setting insurance rates. It is the foundation in which the methods of inferential statistics are built.

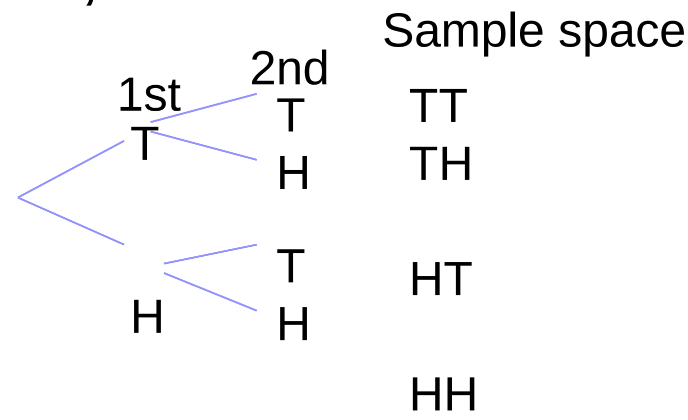
4-1 Sample Spaces and Probability

- A **probability experiment** is a chance process that leads to well-defined results called outcomes.
- An **outcome** is the result of a single trial of a probability experiment.
- A **sample space** is the set of all possible outcomes of a probability experiment.
- An **event** (A, B, C) consists of outcomes
(simple event: 1 outcome, compound event: more than one event)

Sample Spaces

Experiment	Sample Space
Toss a coin	Head, Tail
Roll a die	1, 2, 3, 4, 5, 6
Answer a true/false question	True, False
Toss two coins	HH, HT, TH, TT

Tree diagram



2 X 2 = 4

Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-1

Page #184

Example 4-1: Rolling Dice

Find the sample space for rolling two dice.

6 * 6 = 36

Die 1	Die 2					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-3

Page #184

Example 4-3: Gender of Children

Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl.

BBB BBG BGB BGG GBB GBG GGB GGG

Chapter 4
Probability and Counting Rules

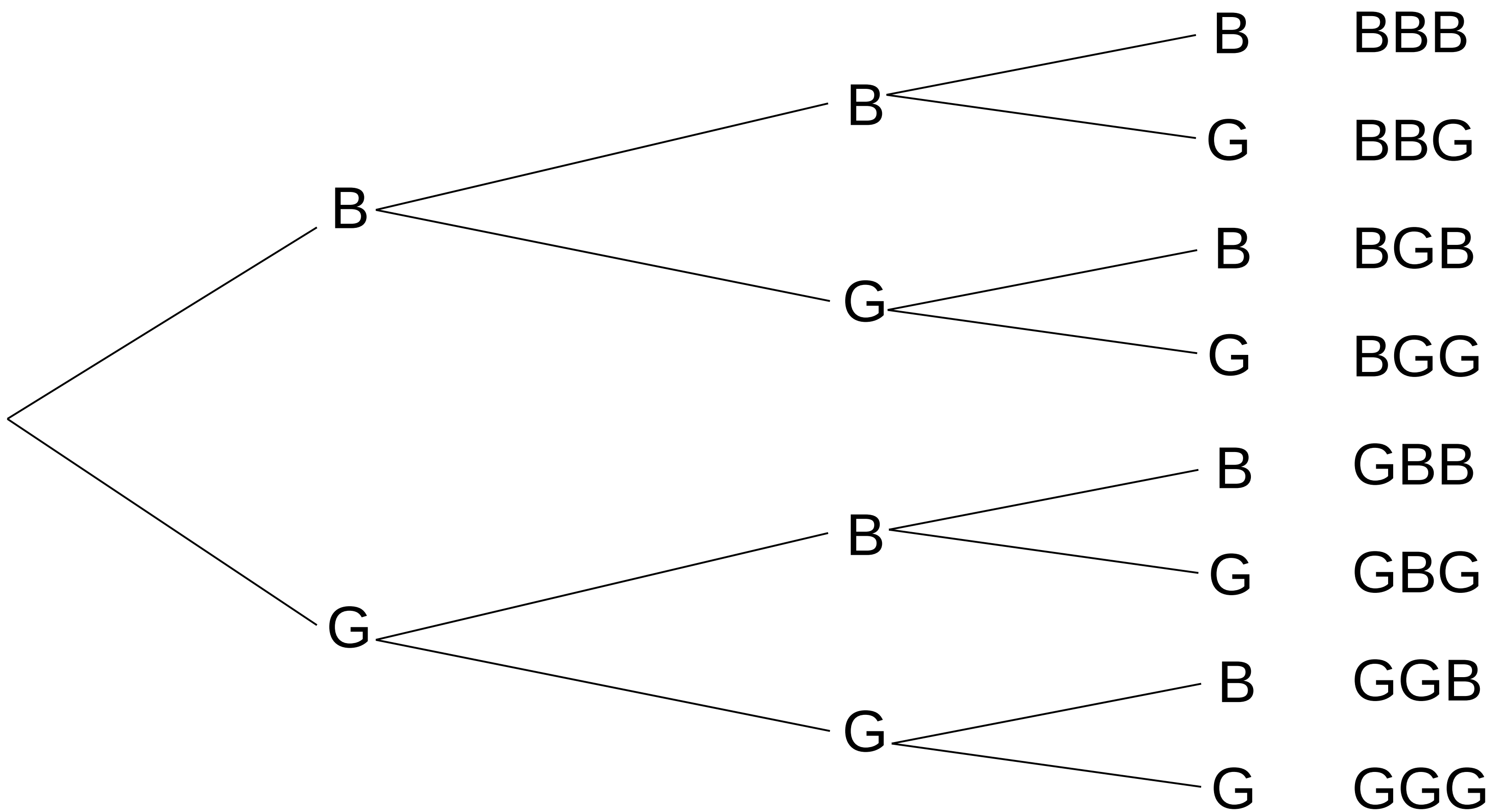
Section 4-1

Example 4-4

Page #185

Example 4-4: Gender of Children

Use a tree diagram to find the sample space for the gender of three children in a family.



Sample Spaces and Probability

There are three basic interpretations of probability:

- **Classical probability**
- **Empirical probability**
- **Subjective probability**

Sample Spaces and Probability

Classical probability uses sample spaces to determine the numerical probability that an event will happen and assumes that all outcomes in the sample space are equally likely to occur. (Requires equally likely outcomes)

$$\boxed{P(E)=\frac{n(E)}{n(S)}=\frac{\text{\# of desired outcomes}}{\text{Total \# of possible outcomes}}}$$

Sample Spaces and Probability

Rounding Rule for Probabilities

Probabilities should be expressed as reduced fractions or rounded to two or three decimal places. When the probability of an event is an extremely small decimal, it is permissible to round the decimal to the first nonzero digit after the decimal point. For example, 0.0000587 would be 0.00006.

There are four basic probability rules.

Probability Rule 1

The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \leq P(E) \leq 1$.

Probability Rule 2

If an event E cannot occur (i.e., the event contains no members in the sample space), its probability is 0.

Probability Rule 3

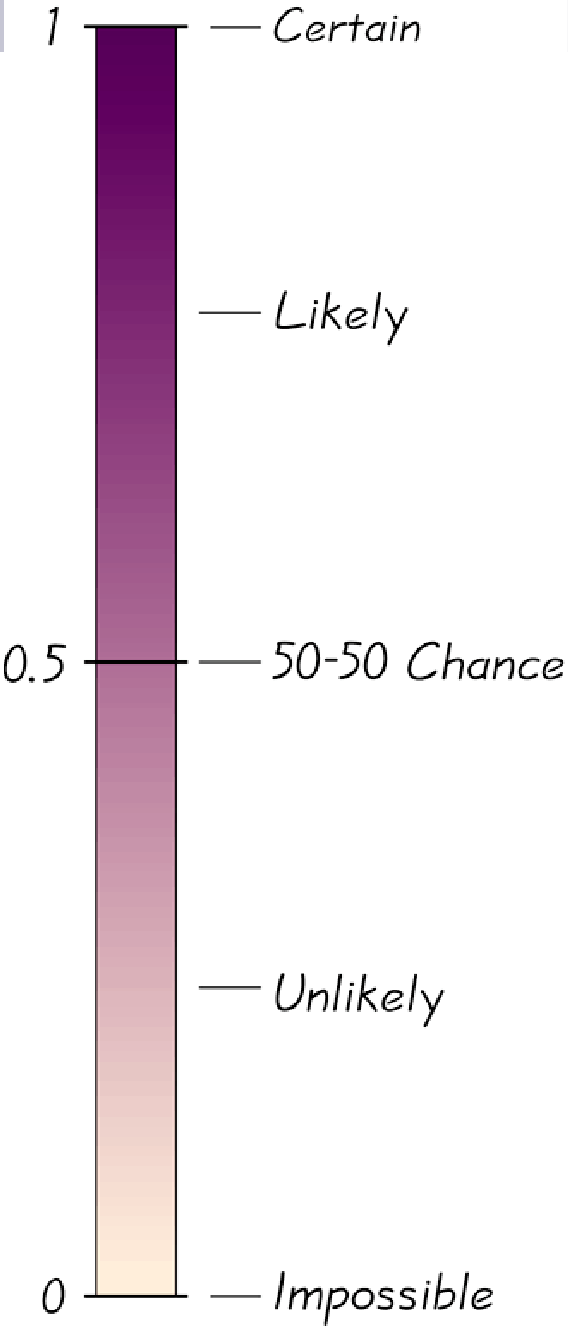
If an event E is certain, then the probability of E is 1.

Probability Rule 4

Outcome	1		2		3		4		5		6
Probability	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$
Sum	$\frac{1}{6}$	+	$\frac{1}{6}$	+	$\frac{1}{6}$	+	$\frac{1}{6}$	+	$\frac{1}{6}$	+	$\frac{1}{6} = \frac{6}{6} = 1$

The sum of the probabilities of all the outcomes in the sample space is 1.

Possible Values for Probabilities



Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-6

Page #187

Example 4-6: Gender of Children

If a family has three children, find the probability that two of the three children are girls.

Sample Space:

BBB BBG BGB BGG GBB GBG GGB GGG

Three outcomes (BGG, GBG, GGB) have two girls.

The probability of having two of three children being girls is 3/8.

Chapter 4
Probability and Counting Rules

Section 4-1

Exercise 4-13c

Page #196

Exercise 4-13c: Rolling Dice

If two dice are rolled one time, find the probability of getting a sum of 7 or 11.

Die 1	Die 2					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$P(\text{sum of 7 or 11}) = \frac{6+2}{36} = \boxed{\frac{2}{9}}$$

Sample Spaces and Probability

The ***complement of an event*** **A**, denoted by $\bar{A}$, is the set of outcomes in the sample space that are not included in the outcomes of event **A**.

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-10

Page #189

Example 4-10: Finding Complements

Find the complement of each event.

Event	Complement of the Event
Rolling a die and getting a 4	Getting a 1, 2, 3, 5, or 6
Selecting a letter of the alphabet and getting a vowel	Getting a consonant (assume y is a consonant)
Selecting a month and getting a month that begins with a J	Getting February, March, April, May, August, September, October, November, or December
Selecting a day of the week and getting a weekday	Getting Saturday or Sunday
Rain	?

Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-11

Page #190

Example 4-11: Residence of People

If the probability that a person lives in an industrialized country of the world is $\frac{1}{5}$, find the probability that a person does not live in an industrialized country.

$$P(\text{Not living in industrialized country})$$
$$= 1 - P(\text{living in industrialized country})$$
$$= 1 - \frac{1}{5} = \boxed{\frac{4}{5}}$$

Sample Spaces and Probability

There are three basic interpretations of probability:

- **Classical probability**
- **Empirical probability**
- **Subjective probability**

Sample Spaces and Probability

Empirical probability (Relative frequency approximation of probability) relies on actual experience (observation) to determine the likelihood of outcomes.

$$\boxed{P(E) = \frac{f}{n} = \frac{\text{frequency of desired class}}{\text{Sum of all frequencies}}}$$

Chapter 4
Probability and Counting Rules

Section 4-1

Example 4-13

Page #192

Example 4-13: Blood Types

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

Type	Frequency
A	22
B	5
AB	2
O	21
	Total 50

$$\boxed{P(\mathrm{O}) = \frac{f}{n} = \frac{21}{50}}$$

Example 4-13: Blood Types

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

b. A person has type A or type B blood.

Type	Frequency
A	22
B	5
AB	2
O	21
	Total 50

$$P(\text{A or B}) = \frac{22}{50} + \frac{5}{50} = \frac{27}{50}$$

Example 4-13: Blood Types

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

c. A person has neither type A nor type O blood.

Type	Frequency
A	~~22~~
B	5
AB	2
O	~~21~~
	Total 50

$$P(\text{neither A nor O}) = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$$

Example 4-13: Blood Types

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

d. A person does not have type AB blood.

Type	Frequency
A	22
B	5
AB	2
O	21
	Total 50

$$P(\text{not AB})$$
$$= 1 - P(\text{AB})$$
$$= 1 - \frac{2}{50} = \frac{48}{50} = \frac{24}{25}$$

Sample Spaces and Probability

There are three basic interpretations of probability:

- **Classical probability**
- **Empirical probability**
- **Subjective probability**

Sample Spaces and Probability

Subjective probability uses a probability value based on an educated guess or estimate, employing opinions and inexact information, or by using knowledge of the relevant circumstances.

Examples: weather forecasting, predicting outcomes of sporting events

4.2 Addition Rules for Probability

- Two events are **mutually exclusive events (disjoint)** if they cannot occur at the same time (i.e., they have no outcomes in common)

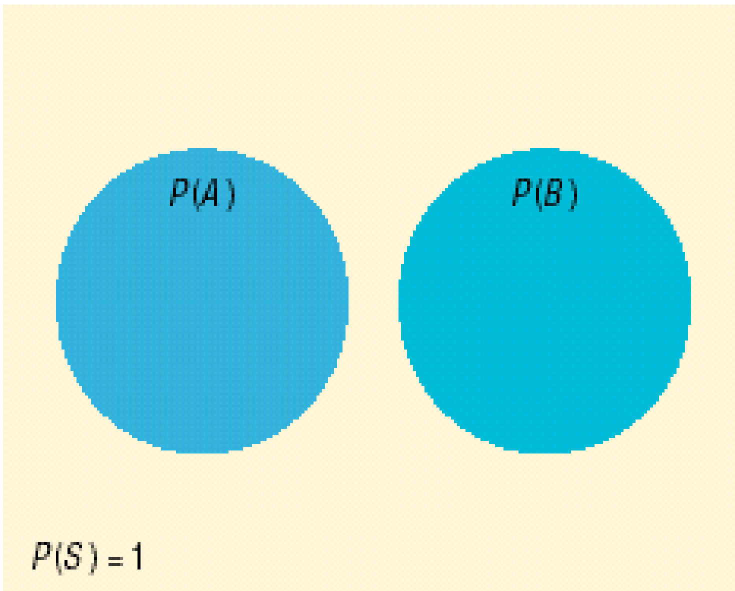
Addition Rules

$$P(A \text{ or } B) = P(A) + P(B) \quad \text{Mutually Exclusive}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \text{Not M. E.}$$

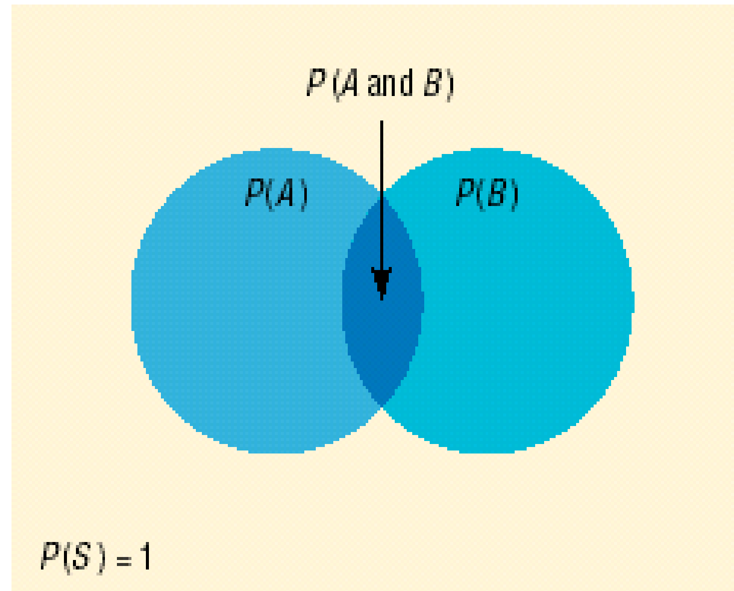
P(A or B) = P(in a single trial, event *A* occurs or event *B* occurs or they both occur).

The key word in this section is "or." It is the *inclusive or*, which means either one or the other or both.



(a) Mutually exclusive events (Disjoint)

$P(A \text{ or } B) = P(A) + P(B)$



(b) Non-mutually exclusive events (Joint)

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Chapter 4
Probability and Counting Rules

Section 4-2

Example 4-15

Page #200

Example 4-15: Rolling a Die

Determine which events are mutually exclusive and which are not, when a single die is rolled.

a. Getting an odd number and getting an even number

Getting an odd number: 1, 3, or 5

Getting an even number: 2, 4, or 6

Mutually Exclusive

Example 4-15: Rolling a Die

Determine which events are mutually exclusive and which are not, when a single die is rolled.

b. Getting a 3 and getting an odd number

Getting a 3: 3

Getting an odd number: 1, 3, or 5

Not Mutually Exclusive

Example 4-15: Rolling a Die

Determine which events are mutually exclusive and which are not, when a single die is rolled.

c. Getting an odd number and getting a number less than 4

Getting an odd number: 1, 3, or 5

Getting a number less than 4: 1, 2, or 3

Not Mutually Exclusive

Example 4-15: Rolling a Die

Determine which events are mutually exclusive and which are not, when a single die is rolled.

d. Getting a number greater than 4 and getting a number less than 4

Getting a number greater than 4: 5 or 6

Getting a number less than 4: 1, 2, or 3

Mutually Exclusive

Chapter 4
Probability and Counting Rules

Section 4-2

Example 4-18

Page #201

Example 4-18: Political Affiliation

At a political rally, there are 20 Republicans, 13 Democrats, and 6 Independents. If a person is selected at random, find the probability that he or she is either a Democrat or a Republicans.

Mutually Exclusive Events

$$P(\text{Democrat or Republican})$$

$$= P(\text{Democrat}) + P(\text{Republican})$$

$$= \frac{13}{39} + \frac{20}{39} = \frac{33}{39} = \boxed{\frac{11}{13}}$$

Chapter 4
Probability and Counting Rules

Section 4-2

Example 4-21

Page #202

Example 4-21: Medical Staff

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male. Not Mutually Exclusive Events

Staff	Females	Males	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

$$P(\text{Nurse or Male}) = P(\text{Nurse}) + P(\text{Male}) - P(\text{Male Nurse})$$
$$= \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \boxed{\frac{10}{13}}$$

4.3 Multiplication Rules

- Two events *A* and *B* are **independent events** if the fact that *A* occurs does not affect the probability of *B* occurring.

examples of independent events:

Rolling a die and getting a 6, and then rolling a second die and getting a 3.

Drawing a card from a deck and getting a queen, replacing it, and drawing a second card and getting a queen. P(Queen)= 4/52 = 1/13

Repetitions are allowed.

Multiplication Rules

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad \textbf{Independent}$$

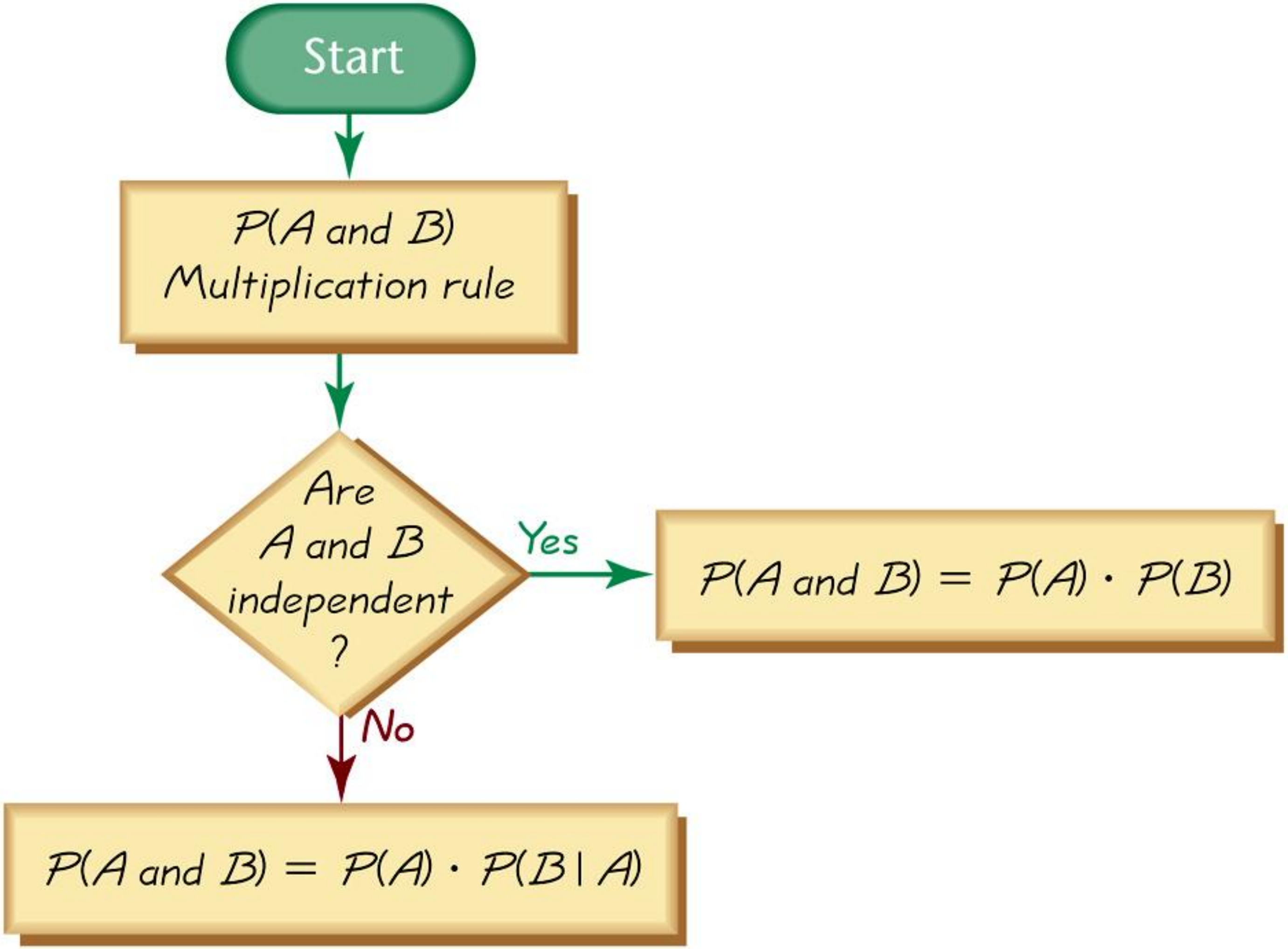
$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad \textbf{Dependent}$$

P(A and B) = P(event A occurring on the first trial and event B occurring on the second trial.

The 5% Guideline for Cumbersome Calculations

If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

Applying the Multiplication Rule



Chapter 4
Probability and Counting Rules

Section 4-3

Example 4-23

Page #211

Example 4-23: Tossing a Coin

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

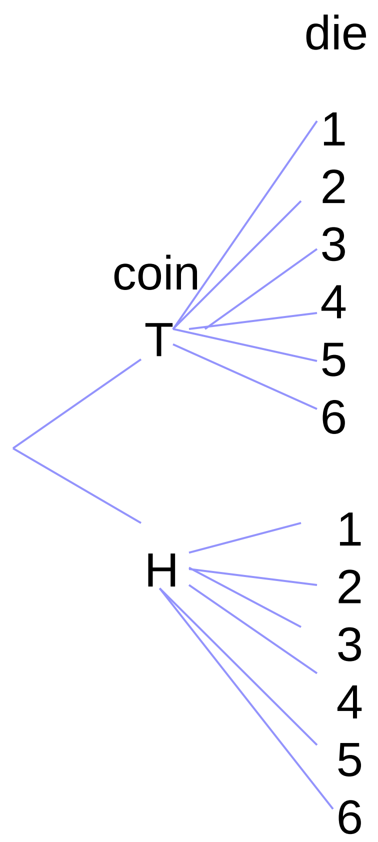
Independent Events

$$P(\text{Head and } 4) = P(\text{Head}) \cdot P(4)$$
$$= \frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{1}{12}}$$

This problem could be solved using sample space.

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.



Sample space: 12	P(H & 4)
T1	
T2	
T3	
T4	
T5	
T6	
H1	
H2	
H3	
H4	1/12
H5	
H6	

2 X 6 = 12 possible combinations

Chapter 4
Probability and Counting Rules

<u>Section 4-3</u>

Example 4-26

Page #212

Example 4-26: Survey on Stress

A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.

Independent Events

$$
\begin{aligned}
P(\mathrm{S \text{ and } S \text{ and } S}) &= P(\mathrm{S}) \cdot P(\mathrm{S}) \cdot P(\mathrm{S}) \\
&= (0.46)(0.46)(0.46) \\
&= \boxed{0.097}
\end{aligned}
$$

Chapter 4
Probability and Counting Rules

Section 4-3

Example 4-28

Page #214

Example 4-28: University Crime

At a university in western Pennsylvania, there were 5 burglaries reported in 2003, 16 in 2004, and 32 in 2005. If a researcher wishes to select at random two burglaries to further investigate, find the probability that both will have occurred in 2004.

(Two distinct case, the first case is selected and not replaced)

n = 5 + 16 + 32 = 53

Dependent Events

$$P(\mathrm{C}_1 \text{ and } \mathrm{C}_2) = P(\mathrm{C}_1) \cdot P(\mathrm{C}_2 | \mathrm{C}_1)$$

$$= \frac{16}{53} \cdot \frac{15}{52} = \frac{60}{689}$$

4.3 Conditional Probability

- **Conditional probability** is the probability that the second event *B* occurs given that the first event *A* has occurred.

Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Chapter 4
Probability and Counting Rules

Section 4-3

Example 4-33

Page #217

Example 4-33: Parking Tickets

The probability that Sam parks in a no-parking zone *and gets a parking ticket is 0.06,* and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.20. On Tuesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

N= parking in a no-parking zone, T= getting a ticket

$$P(\text{T}|\text{N}) = \frac{P(\text{N and T})}{P(\text{N})} = \frac{0.06}{0.20} = \boxed{0.30}$$

Chapter 4
Probability and Counting Rules

Section 4-3

Example 4-34

Page #217

Example 4-34: Women in the Military

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Example 4-34: Women in the Military

a. Find the probability that the respondent answered yes (Y), given that the respondent was a female (F).

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

$$P(\mathrm{Y}|\mathrm{F})=\frac{P(\mathrm{F\ and\ Y})}{P(\mathrm{F})}=\frac{\frac{8}{100}}{\frac{50}{100}}=\frac{8}{50}=\boxed{\frac{4}{25}}$$

Example 4-34: Women in the Military

b. Find the probability that the respondent was a male (M), given that the respondent answered no (N).

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

$$P(\mathrm{M}|\mathrm{N}) = \frac{P(\mathrm{N\ and\ M})}{P(\mathrm{N})} = \frac{\frac{18}{100}}{\frac{60}{100}} = \frac{18}{60} = \boxed{\frac{3}{10}}$$

Complements: The Probability of "At Least One"

- "At least one" is equivalent to "one or more."

- **The complement of getting at least one item of a particular type is that you get no items of that type.**

  P(at least one) = 1 – P(none).

Chapter 4
Probability and Counting Rules

Section 4-3

Example 4-37

Page #219

Example 4-37: Bow Ties

The Neckware Association of America reported that 3% of ties sold in the United States are bow ties (B). If 4 customers who purchased a tie are randomly selected, find the probability that at least 1 purchased a bow tie.

$$P(\mathrm{B}) = 0.03,\ P(\bar{\mathrm{B}}) = 1 - 0.03 = 0.97$$

$$P(\text{no bow ties}) = P(\bar{\mathrm{B}}) \cdot P(\bar{\mathrm{B}}) \cdot P(\bar{\mathrm{B}}) \cdot P(\bar{\mathrm{B}})$$
$$= (0.97)(0.97)(0.97)(0.97) = 0.885$$

Notes: at least one is complement of none of them

$$P(\text{at least 1 bow tie}) = 1 - P(\text{no bow ties})$$
$$= 1 - 0.885 = \boxed{0.115}$$

In short: P(at least 1 bow tie) = 1 – $(1 - .03)^4$ = .115

4.4 Counting Rules

- The **fundamental counting rule** is also called the **multiplication of choices**.

- In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdots k_n$$

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-39

Page #225

Example 4-39: Paint Colors

A paint manufacturer wishes to manufacture several different paints. The categories include

Color: red, blue, white, black, green, brown, yellow	7
Type: latex, oil	2
Texture: flat, semigloss, high gloss	3
Use: outdoor, indoor	2

How many different kinds of paint can be made if you can select one color, one type, one texture, and one use?

$$\begin{pmatrix}\text{\# of}\\ \text{colors}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{types}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{textures}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{uses}\end{pmatrix}$$

$$7 \cdot 2 \cdot 3 \cdot 2$$

$$\boxed{\text{84 different kinds of paint}}$$

Counting Rules

- **Factorial** is the product of all the positive numbers from 1 to a number.

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

$$0! = 1$$ By special definition

- **Permutation** is an arrangement of objects in a specific order. Order matters.

$$_nP_r = \frac{n!}{(n-r)!} = \underbrace{n(n-1)(n-2)\cdots(n-r+1)}_{r \text{ items}}$$

$_6P_3$ = 6! / (6-3)! = 6*5*4*3*2*1 / 3*2*1 = 6*5*4 = 120

(3 items)

Counting Rules

- **Combination** is a grouping of objects. Order does not matter.

$$ {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

$$ = \frac{{}_nP_r}{r!} $$

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-42/4-43

Page #228

Example 4-42: Business Locations

Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?

$$\begin{pmatrix}\text{first}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{second}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{third}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{fourth}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{fifth}\\\text{choice}\end{pmatrix}$$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\boxed{\text{120 different ways to rank the locations}}$$

Using factorials, 5! = 120.

Using permutations, ${}_5P_5 = 5! / (5 – 5) ! = 120$.

Example 4-43: Business Locations

Suppose the business owner in Example 4–42 wishes to rank only the top 3 of the 5 locations. How many different ways can she rank them?

$$\begin{pmatrix}\text{first}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{second}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{third}\\\text{choice}\end{pmatrix}$$

$$5 \quad \cdot \quad 4 \quad \cdot \quad 3$$

$$\boxed{\text{60 different ways to rank the locations}}$$

Using permutations, ${}_5P_3 = 60$.

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-44

Page #229

Example 4-44: Television News Stories

A television news director wishes to use 3 news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of 8 stories to choose from, how many possible ways can the program be set up?

Since there is a lead, second, and closing story, we know that order matters. We will use permutations.

$${}_8P_3 = \frac{8!}{5!} = 336 \qquad \text{or} \qquad {}_8P_3 = 8 \cdot \underset{3}{7} \cdot 6 = 336$$

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-45

Page #229

Example 4-45: School Musical Plays

A school musical director can select 2 musical plays to present next year. One will be presented in the fall, and one will be presented in the spring. If she has 9 to pick from, how many different possibilities are there?

Order matters, so we will use permutations.

$$ {}_9P_2 = \frac{9!}{7!} = 72 \quad \text{or} \quad {}_9P_2 = \underbrace{9 \cdot 8}_{2} = 72 $$

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-48

Page #231

Example 4-48: Book Reviews

A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

The placement in the newspaper is not mentioned, so order does not matter. We will use combinations.

$$ {}_nC_r = \frac{n!}{(n-r)!r!} \qquad {}_8C_3 = \frac{8!}{5!3!} = 8!/(5!3!) = 56 $$

$$ \text{or} \quad {}_8C_3 = \frac{8\cdot 7\cdot 6}{3\cdot 2} = 56 \qquad \text{or} \quad {}_8C_3 = \frac{{}_8P_3}{3!} = 56 \qquad = \frac{{}_nP_r}{r!} $$

Chapter 4
Probability and Counting Rules

Section 4-4

Example 4-49

Page #231

Example 4-49: Committee Selection

In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

There are not separate roles listed for each committee member, so order does not matter. We will use combinations.

$$\text{Women: } {}_7C_3 = \frac{7!}{4!3!} = 35, \quad \text{Men: } {}_5C_2 = \frac{5!}{3!2!} = 10$$

There are 35·10=350 different possibilities.

4.5 Probability and Counting Rules

The counting rules can be combined with the probability rules in this chapter to solve many types of probability problems.

By using the fundamental counting rule, the permutation rules, and the combination rule, you can compute the probability of outcomes of many experiments, such as getting a full house when 5 cards are dealt or selecting a committee of 3 women and 2 men from a club consisting of 10 women and 10 men.

Chapter 4
Probability and Counting Rules

Section 4-5

Example 4-52

Page #238

Example 4-52: Committee Selection

A store has 6 *TV Graphic magazines and 8 Newstime* magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

TV Graphic: One magazine of the 6 magazines

Newstime: One magazine of the 8 magazines

Total: Two magazines of the 14 magazines

$$\frac{{}_6C_1 \cdot {}_8C_1}{{}_{14}C_2} = \frac{6 \cdot 8}{91} = \boxed{\frac{48}{91}}$$

Chapter 4
Probability and Counting Rules

<u>Section 4-5</u>

Example 4-53

Page #239

Example 4-53: Combination Locks

A combination lock consists of the 26 letters of the alphabet. If a 3-letter combination is needed, find the probability that the combination will consist of the letters ABC in that order. The same letter can be used more than once. (*Note: A combination lock is really a* permutation lock.)

There are 26·26·26 = 17,576 possible combinations.

The letters ABC in order create one combination.

$$\boxed{P(\text{ABC}) = \frac{1}{17{,}576}}$$