Hypothesis Testing

Objectives

After completing this chapter, you should be able to

1 Understand the definitions used in hypothesis testing.

2 State the null and alternative hypotheses.

3 Find critical values for the z test.

4 State the five steps used in hypothesis testing.

5 Test means when σ is known, using the z test.

6 Test means when σ is unknown, using the t test.

7 Test proportions, using the z test.

8 Test variances or standard deviations, using the chi-square test.

9 Test hypotheses, using confidence intervals.

10 Explain the relationship between type I and type II errors and the power of a test.

Outline

Review

In Chapters 2 and 3 we used "descriptive statistics" when we summarized data using tools such as graphs, and statistics such as the mean and standard deviation. Methods of inferential statistics use sample data to make an inference or conclusion about a population. The two main activities of inferential statistics are using sample data to (1) estimate a population parameter (such as estimating a population parameter with a confidence interval), and (2) test a hypothesis or claim about a population parameter.

In Chapter 7 we presented methods for estimating a population parameter with a confidence interval,

and in this chapter we present the method of hypothesis testing.

Hypothesis Testing

Researchers are interested in answering many types of questions. For example,

- ☐ Is the earth warming up?

- ☐ Does a new medication lower blood pressure?

- ☐ Does the public prefer a certain color in a new fashion line?

- ☐ Is a new teaching technique better than a traditional one?

- ☐ Do seat belts reduce the severity of injuries?

These types of questions can be addressed through statistical hypothesis testing, which is a decision-making process for evaluating claims about a population.

A hypothesis is a claim or statement about a property of a population (μ, σ and p).

Why need hypothesis testing?

How Much Better Is Better?

Suppose a school superintendent reads an article which states that the overall mean score for the SAT is 910. Furthermore, suppose that, for a sample of students, the average of the SAT scores in the superintendent's school district is 960. Can the superintendent conclude that the students in his school district scored higher than average? At first glance, you might be inclined to say yes, since 960 is higher than 910. But recall that the means of samples vary about the population mean when samples are selected from a specific population. So the question arises, Is there a real difference in the means, or is the difference simply due to chance (i.e., sampling error)? In this chapter, you will learn how to answer that question by using statistics that explain hypothesis testing. See Statistics Today—Revisited for the answer. In this chapter, you will learn how to answer many questions of this type by using statistics that are explained in the theory of hypothesis testing.

Hypothesis Testing

Three methods used to test hypotheses:

1. The traditional method

2. The P-value method

3. The confidence interval method

8.1 Steps in Hypothesis Testing-Traditional Method

- A **statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.

- The **null hypothesis**, symbolized by H_0 (H_n or H_N), is a statistical hypothesis that states that there is <u>no difference</u> between a parameter and a specific value, or that there is <u>no difference</u> between two parameters, use =.
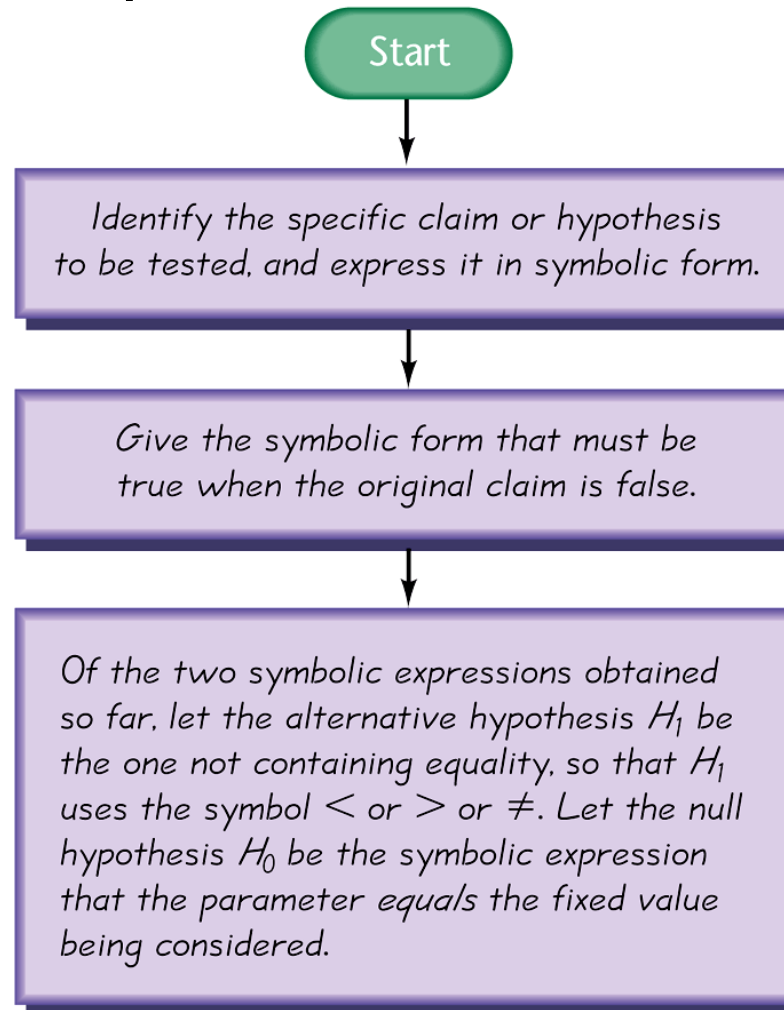
Steps in Hypothesis Testing-Traditional Method

- The **alternative hypothesis**, symbolized by H_1 (H_a or H_A), is a statistical hypothesis that states the existence of a <u>difference</u> between a parameter and a specific value, or states that there is a <u>difference</u> between two parameters. The symbolic form of the alternative hypothesis must use one of these symbols: $\neq, <, >$.

Note about Forming Your Own Claims (Hypotheses)

If you are conducting a study and want to use a hypothesis test to support your claim, the claim must be worded so that it becomes the alternative hypothesis.

Note about Identifying H_0 and H_1

Start

Identify the specific claim or hypothesis to be tested, and express it in symbolic form.

Give the symbolic form that must be true when the original claim is false.

Of the two symbolic expressions obtained so far, let the alternative hypothesis H_1 be the one not containing equality, so that H_1 uses the symbol $<$ or $>$ or $\neq$. Let the null hypothesis H_0 be the symbolic expression that the parameter equals the fixed value being considered.

The null and alternative hypotheses are stated together, and the null hypothesis contains the equals sign, as shown (where k represents a specified number).

One-tailed test

	Two-tailed test	Right-tailed test	Left-tailed test
By H_1			
No difference →	$H_0: \mu = k$	$H_0: \mu = k$	$H_0: \mu = k$ _with equal sign_
Researcher's interest →	$H_1: \mu \neq k$	$H_1: \mu > k$	$H_1: \mu < k$

Note: In this book, the null hypothesis is always stated using the equals sign

There is no overlapping between H_0 and H_1

Situation A

A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication. Will the pulse rate <u>increase, decrease</u>, or remain unchanged after a patient takes the medication? The researcher knows that the mean pulse rate for the population under study is 82 beats per minute.

The hypotheses for this situation in symbolic form are

$$H_0 : \ \mu = 82 \qquad H_1 : \ \mu \neq 82$$

This is called a **two-tailed** hypothesis test.

Situation B

A chemist invents an additive to increase the life of an automobile battery. The mean lifetime of the automobile battery without the additive is 36 months.

In this book, the null hypothesis is always stated using the equals sign. The hypotheses for this situation are

$$H_0: \; \mu = 36 \qquad H_1: \; \mu > 36$$

This is called a **right-tailed** hypothesis test.

Situation C

A contractor wishes to <u>lower heating bills</u> by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses about heating costs with the use of insulation are

The hypotheses for this situation are

$$H_0: \mu = 78 \qquad H_1: \mu < 78$$

This is called a **left-tailed** hypothesis test.

Claim

When a researcher conducts a study, he or she is generally looking for evidence to support a **claim**. Therefore, the claim should be stated as the alternative hypothesis, or **research hypothesis**.

A claim, though, can be stated as either the null hypothesis or the alternative hypothesis; however, the statistical evidence can only _support_ the claim <u>if it is the alternative hypothesis</u>. Statistical evidence can be used to _reject_ the claim <u>if the claim is the null hypothesis</u>.

These facts are important when you are stating the conclusion of a statistical study.

Hypothesis Testing

A decision-making process for evaluating claims about a population.

The researcher must:

- -Define the population under study,   (e.g. MDC students)

- -State the particular hypotheses that will be investigated,

     (e.g. students who attend class more have better grades)

- -Give the significance level,   (e.g. α = 0.05)

- -Select a sample from the population,

     (e.g. randomly select 2 classes)

- -Collect the data,   (e.g. class attendance, test score)

- -Perform the calculations required for the statistical test,

- -Reach a conclusion.

Hypothesis Testing

hypothesis. The question is, Where does the researcher draw the line? This decision is not made on feelings or intuition; it is made statistically. That is, the difference must be significant and in all likelihood not due to chance. Here is where the concepts of statistical test and level of significance are used.

- A **statistical test** uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected.

- The numerical value obtained from a statistical test is called the **test value**.

- In the hypothesis-testing situation, there are <u>four possible outcomes</u>.

Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

- A **type I error (α error: false positive)** occurs if one rejects the null hypothesis when it is true.

- A **type II error (β error: false negative)** occurs if one does not reject the null hypothesis when it is false.

Hypothesis Testing

	H_0 true	H_0 false
Reject H_0	**Error** Type I (α error, false positive)	Correct decision
Do not reject H_0	Correct decision	**Error** Type II (β error, false negative)

In the hypothesis-testing situation, there are four possible outcomes. In reality, the null hypothesis may or may not be true, and a decision is made to reject or not reject it on the basis of the data obtained from a sample. The four possible outcomes are shown in Figure 8-2. Notice that there are two possibilities for a correct decision and two possibilities for an incorrect decision.

β can not easily be computed, however, α and β are related in that decrease one increases the other.

Power of a Statistical Test

The **power of a test** measures the sensitivity of the test to detect a real difference in parameters if one actually exists. The higher the power, the more sensitive the test. The power is $1 - \beta$.

	H_0 true	H_0 false
Reject H_0	**Type I** error α	Correct decision $1 - \beta$
Do not reject H_0	Correct decision $1 - \alpha$	**Type II** error β

Hypothesis Testing

- The **level of significance** is the maximum probability of committing a type I error and is the probability that the test statistic will fall in the critical region when the null hypothesis is actually true. This probability is symbolized by α (alpha). That is,

$$P(\text{type I error}) = \alpha. \quad \text{Likewise,}$$

$$P(\text{type II error}) = \beta \ (\text{beta}).$$

Hypothesis Testing

- Typical significance levels are:

   0.10, <u>0.05</u>, and 0.01

- For example, when $\alpha = 0.05$, there is a 5% chance of rejecting a true null hypothesis.

Hypothesis Testing

- The **critical value, C.V.**, separates the critical region from the noncritical region.

- The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.

- The **noncritical** or **nonrejection region** is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

A **one-tailed test** indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean. A one-tailed test is either a **right-tailed test** or **left-tailed test,** depending on the direction of the inequality of the alternative hypothesis.

In a **two-tailed test,** the null hypothesis should be rejected when the test value is in either of the two critical regions.

Two-tailed Test

H_0: =

H_1: ≠

α is divided equally between the two tails of the critical region

Means less than or greater than

Sign used in H_1: ≠

Left-tailed Test

$$H_0: =$$

$$H_1: <$$

α the left tail

Points Left

Reject H_0

Fail to reject H_0

Sign used in H_1: <
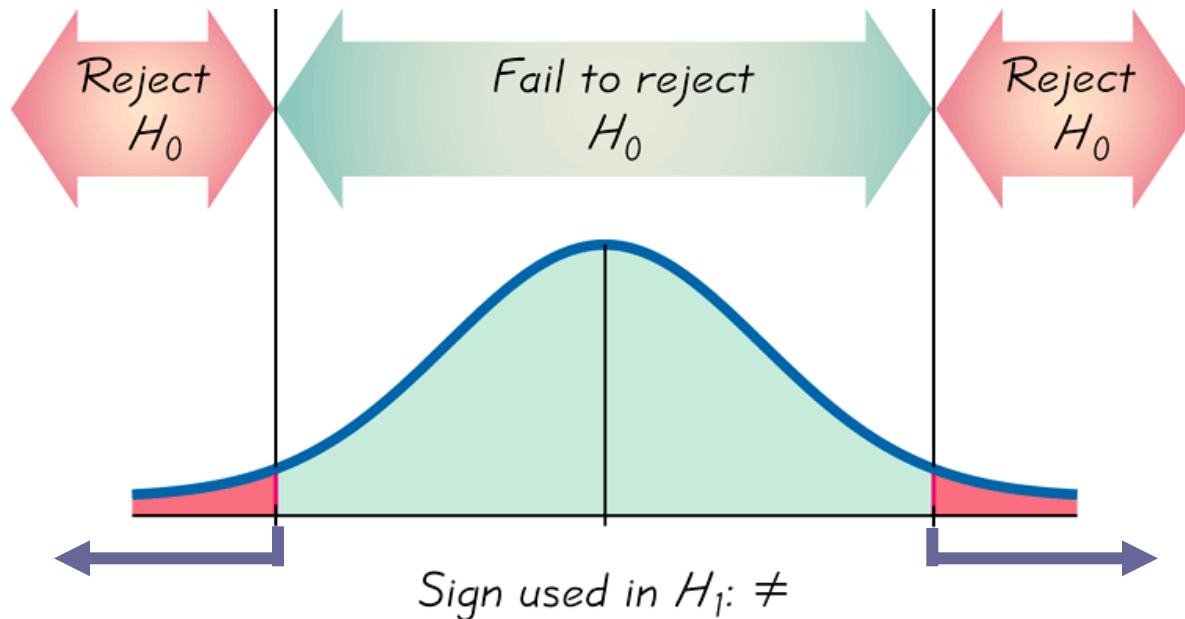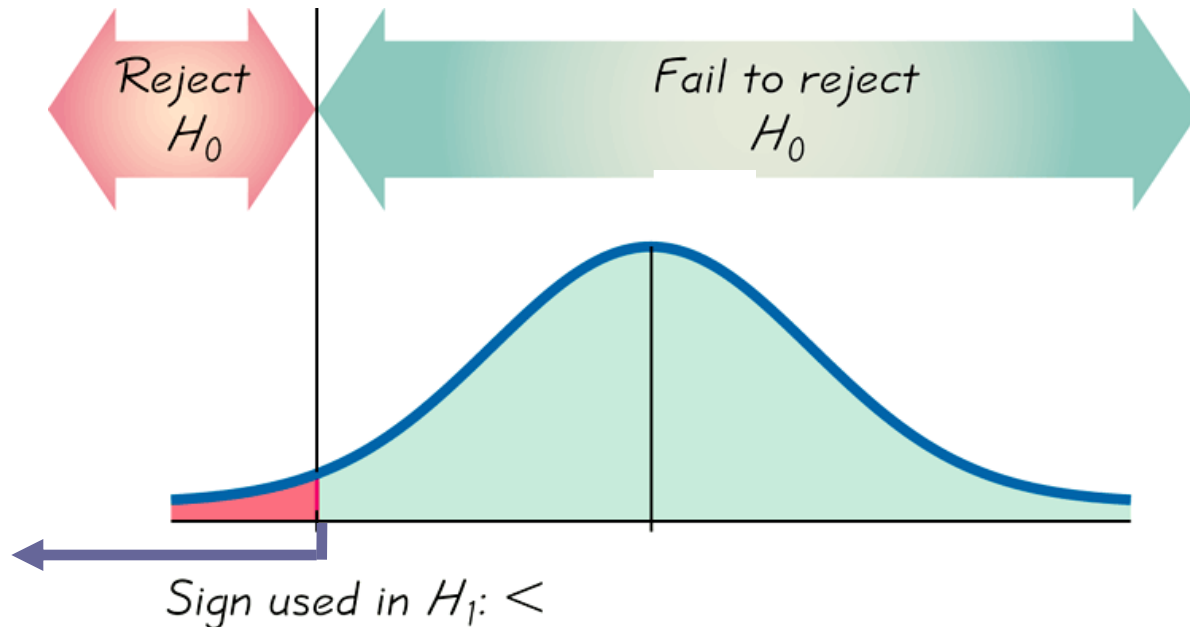
Right-tailed Test

$$H_0: \ =$$

$$H_1: \ >$$

Points Right

Fail to reject H_0

Reject H_0

Sign used in H_1: >
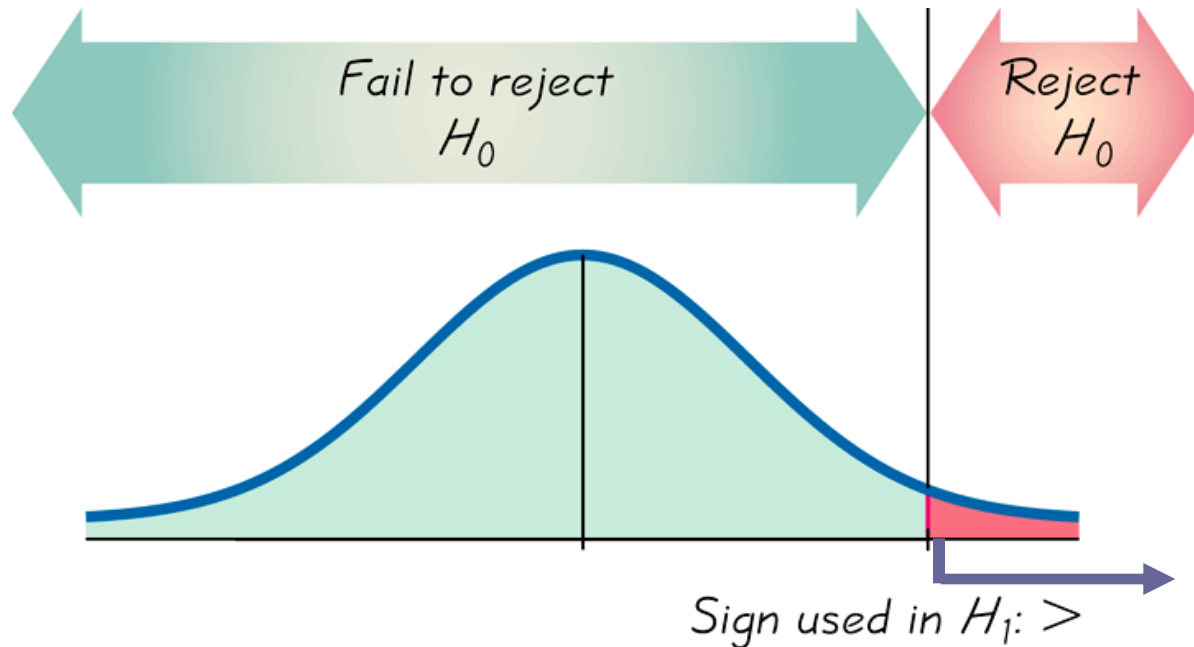
Hypothesis Testing

Finding the Critical Value for $\alpha = 0.01$ (Right-Tailed Test)

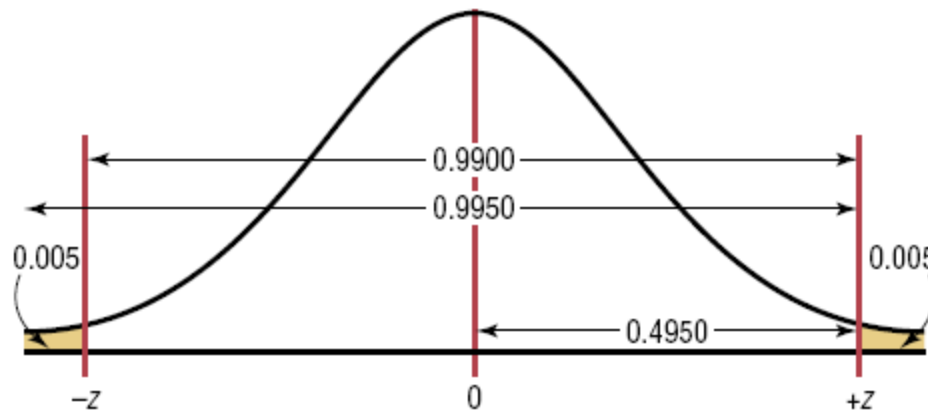$z = 2.33$ for $\alpha = 0.01$ (Right-Tailed Test)

Hypothesis Testing

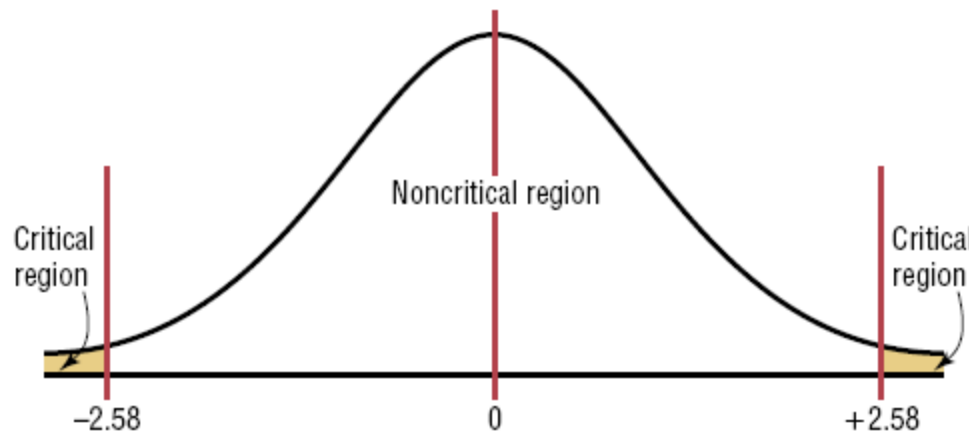Finding the Critical Value for $\alpha = 0.01$ (Left-Tailed Test)
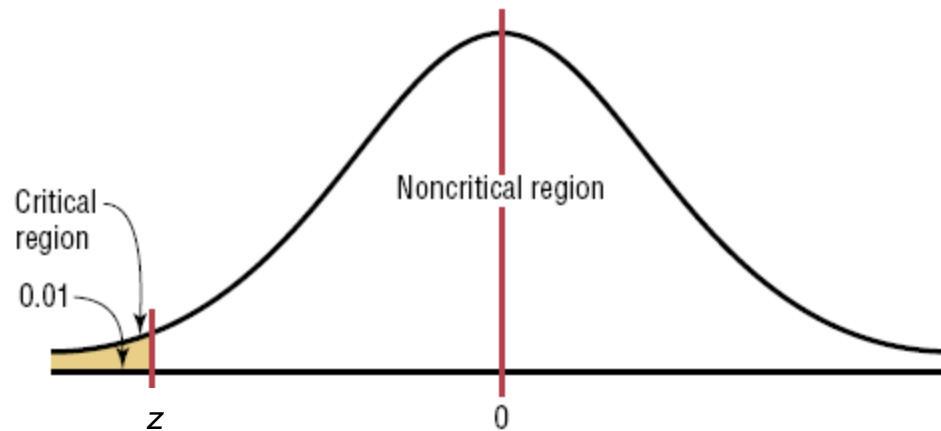
Because of symmetry,
$z = $ -2.33 for $\alpha = 0.01$ (Left-Tailed Test)

Hypothesis Testing

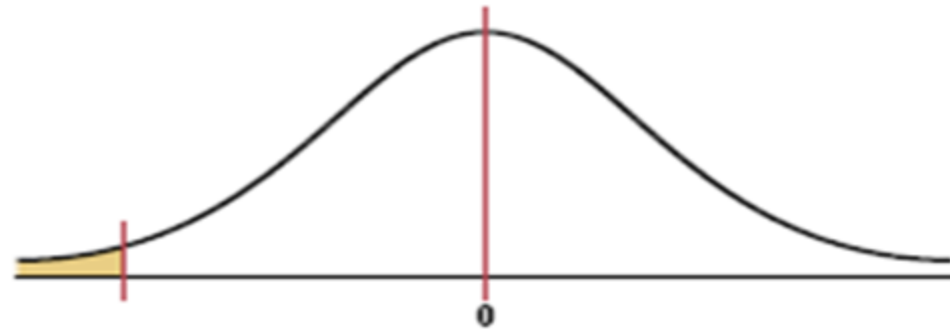Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)
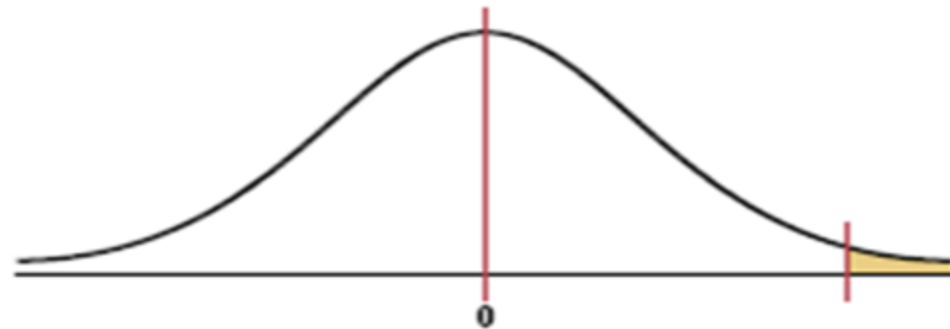

$z = \pm2.58$

$H_0: \mu = k$
$H_1: \mu < k$ $\begin{cases} \alpha = 0.10, \text{ C.V.} = -1.28 \\ \alpha = 0.05, \text{ C.V.} = -1.65 \\ \alpha = 0.01, \text{ C.V.} = -2.33 \end{cases}$

(a) Left-tailed

$H_0: \mu = k$
$H_1: \mu > k$ $\begin{cases} \alpha = 0.10, \text{ C.V.} = +1.28 \\ \alpha = 0.05, \text{ C.V.} = +1.65 \\ \alpha = 0.01, \text{ C.V.} = +2.33 \end{cases}$

(b) Right-tailed

$H_0: \mu = k$
$H_1: \mu \neq k$ $\begin{cases} \alpha = 0.10, \text{ C.V.} = \pm 1.65 \\ \alpha = 0.05, \text{ C.V.} = \pm 1.96 \\ \alpha = 0.01, \text{ C.V.} = \pm 2.58 \end{cases}$

(c) Two-tailed

Procedure Table

Finding the Critical Values for Specific α Values, Using Table E

Step 1 Draw the figure and indicate the appropriate area.

a. If the test is left-tailed, the critical region, with an area equal to α, will be on the left side of the mean.

b. If the test is right-tailed, the critical region, with an area equal to α, will be on the right side of the mean.

c. If the test is two-tailed, α must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.

Procedure Table

Finding the Critical Values for Specific α Values, Using Table E

Step 2 Find the *z* value in Table E.

a. For a left-tailed test, use the *z* value that corresponds to the area equivalent to α in Table E.

b. For a right-tailed test, use the *z* value that corresponds to the area equivalent to $1 - \alpha$.

c. For a two-tailed test, use the *z* value that corresponds to $\alpha / 2$ for the left value. It will be negative. For the right value, use the *z* value that corresponds to the area equivalent to $1 - \alpha / 2$. It will be positive.

Chapter 8
Hypothesis Testing

Section 8-1

Example 8-2

Page #410

Example 8-2: Using Table E

Using Table E in Appendix C, <u>find the critical value(s)</u> for each situation and draw the appropriate figure, showing the critical region.

    a. A <u>left-tailed</u> test with $\alpha = 0.10$.

Step 1 Draw the figure and indicate the appropriate area.

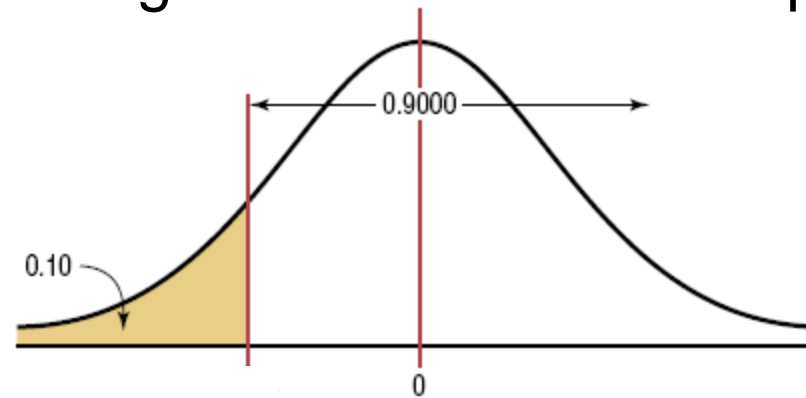**Step 2** Find the area closest to 0.1000 in Table E. In this case, it is 0.1003. The *z* value is -1.28.

Example 8-2: Using Table E

Using Table E in Appendix C, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

b. A <u>two-tailed test</u> with $\alpha = 0.02$.

Step 1 Draw the figure with areas $\alpha / 2 = 0.02/2 = 0.01$.

Step 2 Find the areas closest to 0.01 and 0.99. The areas are 0.0099 and 0.9901. The z values are -2.33 and 2.33.

Example 8-2: Using Table E

Using Table E in Appendix C, find the critical value(s) for each situation and draw the appropriate figure, showing the critical region.

    c. A <u>right-tailed test</u> with $\alpha = 0.005$.

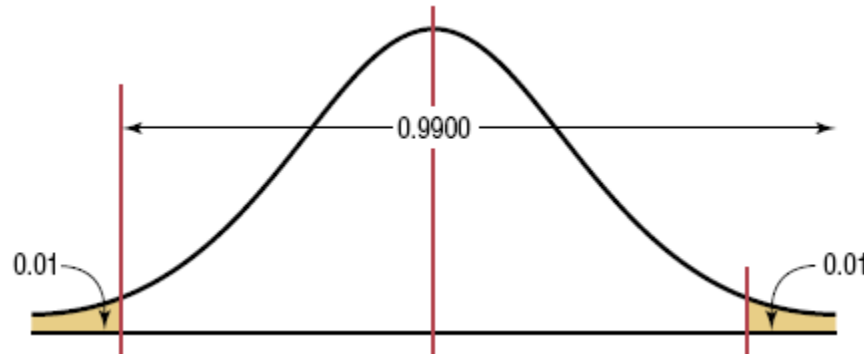**Step 1** Draw the figure and indicate the appropriate area.

0.9950

0.005
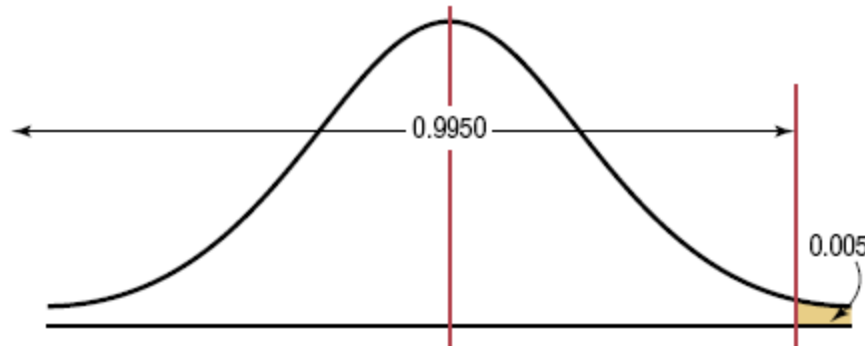
Step 2 Find the area closest to $1 - \alpha = 0.995$. There is a tie: 0.9949 and 0.9951. Average the *z* values of 2.57 and 2.58 to get 2.575 or <u>2.58</u>.

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Find the critical value(s) from the appropriate table in Appendix C.

Step 3 Compute the test value.

Step 4 Make the decision to reject or not reject the null hypothesis.

Step 5 Summarize the results.

8.2 *z* Test for a Mean

The *z* test is a statistical test for the mean of a population. It can be used when *n* ≥ 30, or when the population is normally distributed if n <30 and σ is known.

The formula for the *z test is*

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\overline{X}$ = sample mean

μ = hypothesized population mean

σ = population standard deviation

n = sample size

Chapter 8
Hypothesis Testing

Section 8-2

Example 8-3
Page #414

Example 8-3: Professors' Salaries

A researcher reports that the average salary of assistant professors is more than $42,000. A sample of 30 assistant professors has a mean salary of $43,260. At $\alpha = 0.05$, test the <u>claim</u> that assistant professors earn <u>more than</u> $42,000 per year. The standard deviation of the population is $5230.

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = \$42,000$ *and* H_1: μ > $\$42,000$ (claim)

Step 2: Find the critical value.

Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is $z = 1.65$.

Example 8-3: Professors' Salaries

A researcher reports that the average salary of assistant professors is more than $42,000. A sample of 30 assistant professors has a mean salary of $43,260. At $\alpha = 0.05$, test the claim that assistant professors earn more than $42,000 per year. The standard deviation of the population is $5230.

Step 3: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{43260 - 42000}{5230 / \sqrt{30}} = 1.32$$

Example 8-3: Professors' Salaries

Step 4: Make the decision.

Since the test value, 1.32, is less than the critical value, 1.65, and is not in the critical region, the decision is to not reject the null hypothesis.
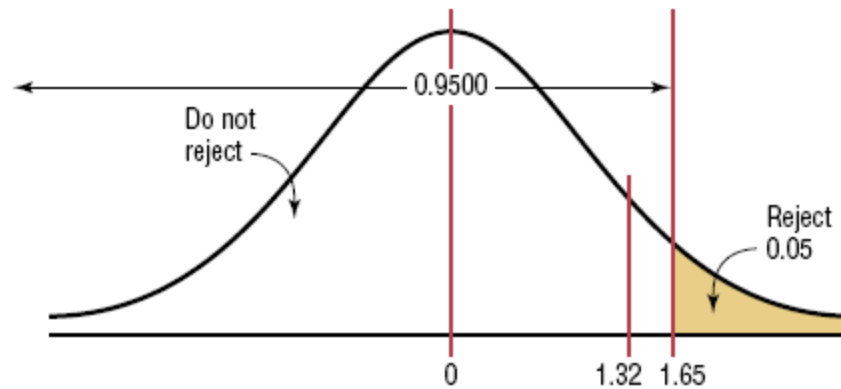
Step 5: Summarize the results.

There is not enough evidence to support the claim that assistant professors earn more on average than $42,000 per year.

Important Comments

- Even though in Example 8–3 the sample mean of $43,260 is higher than the hypothesized population mean of $42,000, it is not significantly higher. Hence, the difference may be due to chance.

- When the null hypothesis is not rejected, there is still a probability of a type II error, i.e., of not rejecting the null hypothesis when it is false.

- When the null hypothesis is not rejected, it cannot be accepted as true. There is merely not enough evidence to say that it is false.

Chapter 8
Hypothesis Testing

Section 8-2

Example 8-4

Page #415

Example 8-4: Cost of Men's Shoes

A researcher <u>claims</u> that the average cost of men's athletic shoes is **<u>less than</u>** $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at $\alpha = 0.10$? Assume $\sigma = 19.2$.

60 70 75 55 80 55   50 40 80 70 50 95

120 90 75 85 80 60 110 65 80 85 85 45

75 60 90 90 60 95 110 85 45 90 70 70

Step 1: State the hypotheses and identify the claim.

$H_0: \mu = \$80$ *and* $H_1: \mu < \$80$ (claim)

Example 8-4: Cost of Men's Shoes

A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at $\alpha = 0.10$? Assume $\sigma = 19.2$.

60 70 75 55 80 55   50 40 80 70 50 95

120 90 75 85 80 60 110 65 80 85 85 45

75 60 90 90 60 95 110 85 45 90 70 70

Step 2: Find the critical value.

Since $\alpha = 0.10$ and the test is a left-tailed test, the critical value is $z = -1.28$.

Example 8-4: Cost of Men's Shoes

A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). (The costs have been rounded to the nearest dollar.) Is there enough evidence to support the researcher's claim at $\alpha = 0.10$? Assume $\sigma = 19.2$.

Step 3: Compute the test value.

Using technology, we find $\bar{X} = 75.0$ and $\sigma = 19.2$.

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{75 - 80}{19.2 / \sqrt{36}} = -1.56$$

Example 8-4: Cost of Men's Shoes

Step 4: Make the decision.

Since the test value, -1.56, falls in the critical region, the decision is to reject the null hypothesis.
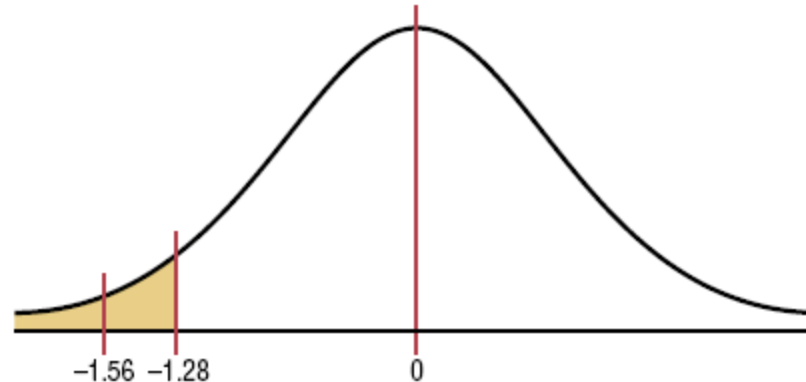
Step 5: Summarize the results.

There is enough evidence to support the claim that the average cost of men's athletic shoes is less than $80.

Chapter 8
Hypothesis Testing

Section 8-2

Example 8-5

Page #416

Example 8-5: Cost of Rehabilitation

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is <u>different from </u>$24,672?

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = \$24,672$ *and* H_1: $\mu \neq \$24,672$ (claim)

Example 8-5: Cost of Rehabilitation

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

Step 2: Find the critical value.

Since $\alpha = 0.01$ and a two-tailed test, the critical values are z = ±2.58.

Example 8-5: Cost of Rehabilitation

"reports that the average cost of rehabilitation for stroke victims is $24,672"

"a random sample of 35 stroke victims"

"the average cost of their rehabilitation is $25,226"

"the standard deviation of the population is $3251"

"α = 0.01"

Step 3: Find the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{25,226 - 24,672}{3251 / \sqrt{35}} = 1.01$$

Example 8-5: Cost of Rehabilitation

Step 4: Make the decision.

Do not reject the null hypothesis, since the test value falls in the noncritical region.
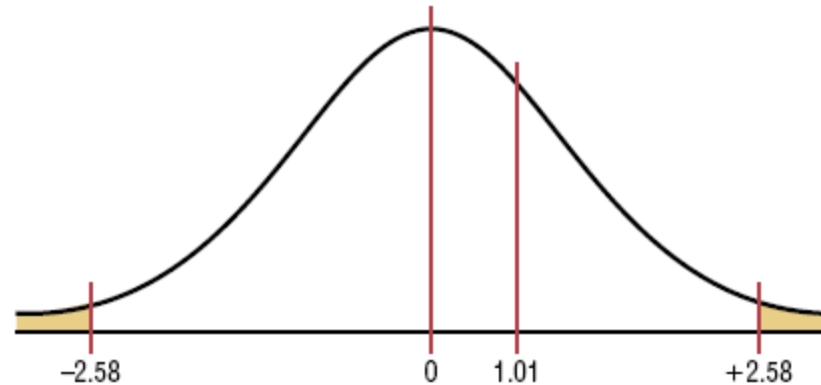
Step 5: Summarize the results.

There is not enough evidence to support the claim that the average cost of rehabilitation at the particular hospital is different from $24,672.

I. Claim is H_0

Reject H_0

There is enough evidence

to reject the claim.

Do not reject H_0

There is not enough evidence

to reject the claim.

(a) Decision when claim is H_0 and H_0 is rejected

II. Claim is H_1

Reject H_0

There is enough evidence

to support the claim.

Do not reject H_0

There is not enough evidence

to support the claim.

(b) Decision when claim is H_1 and H_0 is not rejected

Again, remember that nothing is being proved true or false. The statistician is only stating that there is or is not enough evidence to say that a claim is *probably* true or false. As noted previously, the only way to prove something would be to use the entire population under study, and usually this cannot be done, especially when the population is large.

Hypothesis Testing

The **_P-value_** (or probability value) is the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.

Test Value
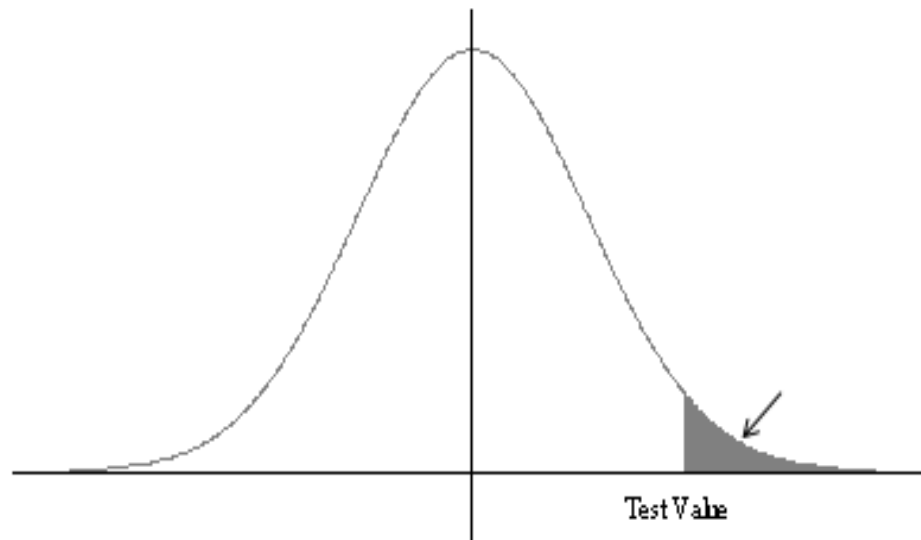
In other words, the _P_-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample statistic or a more extreme sample statistic occurring if the null hypothesis is true.

Hypothesis Testing

- In this section, the traditional method for solving hypothesis-testing problems compares **z-values**:
    - critical value
    - test value

- The *P*-value method for solving hypothesis-testing problems compares **areas**:
    - alpha
    - *P*-value

For example, suppose that a null hypothesis is H_0: μ = 50 and the mean of a sample is $\overline{X}$ = 52. If the computer printed a P-value of 0.0356 for a statistical test, then the probability of getting a sample mean of 52 or greater is 0.0356 if the true population mean is 50 (for the given sample size and standard deviation). The relationship between the P-value and the α value can be explained in this manner. For P = 0.0356, the null hypothesis would be rejected at α = 0.05 but not at α = 0.01. See Figure 8–18.

Area = 0.05

Area = 0.0356

Area = 0.01

50   52
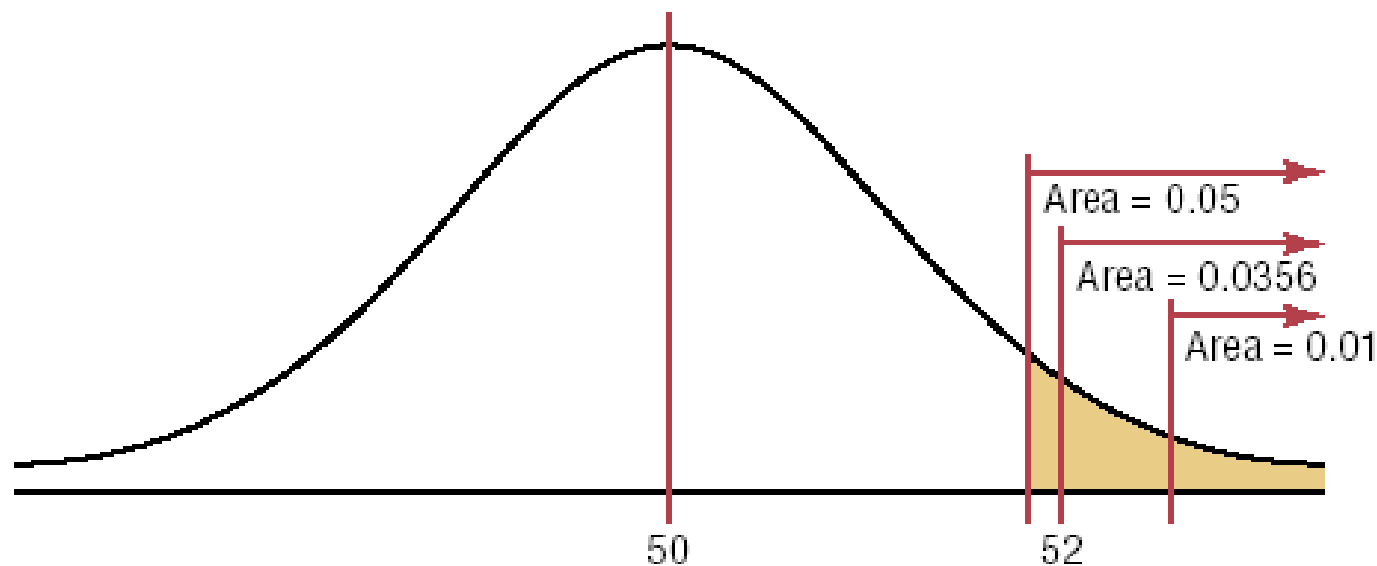
Decision Rule When Using a *P*-Value

If *P*-value $\leq \alpha$, reject the null hypothesis.

If *P*-value $> \alpha$, do not reject the null hypothesis.

Procedure Table

Solving Hypothesis-Testing Problems
(*P*-Value Method)

Step 1 State the hypotheses and identify the claim.

Step 2 Compute the test value.

Step 3 Find the *P*-value.

Step 4 Make the decision.

Step 5 Summarize the results.

Chapter 8
Hypothesis Testing

Section 8-2

Example 8-6

Page #419

Example 8-6: Cost of College Tuition

A researcher wishes to test the <u>claim</u> that the average cost of tuition and fees at a four-year public college is <u>greater than</u> $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? <u>Use the *P*-value method.</u>

Step 1: State the hypotheses and identify the claim.

$H_0: \mu = \$5700$ and $H_1: \mu > \$5700$ (claim)

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? Use the P-value method.

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{5950 - 5700}{659/\sqrt{36}} = 2.28$$

Example 8-6: Cost of College Tuition

A researcher wishes to test the claim that the average cost of tuition and fees at a four-year public college is greater than $5700. She selects a random sample of 36 four-year public colleges and finds the mean to be $5950. The population standard deviation is $659. Is there evidence to support the claim at a 0.05? Use the P-value method.

Step 3: Find the *P*-value.

Using Table E, find the area for z = 2.28.

The area is 0.9887.

Subtract from 1.0000 to find the area of the tail.

Hence, the *P*-value is 1.0000 – 0.9887 = 0.0113.

Example 8-6: Cost of College Tuition

Step 4: Make the decision.

Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.
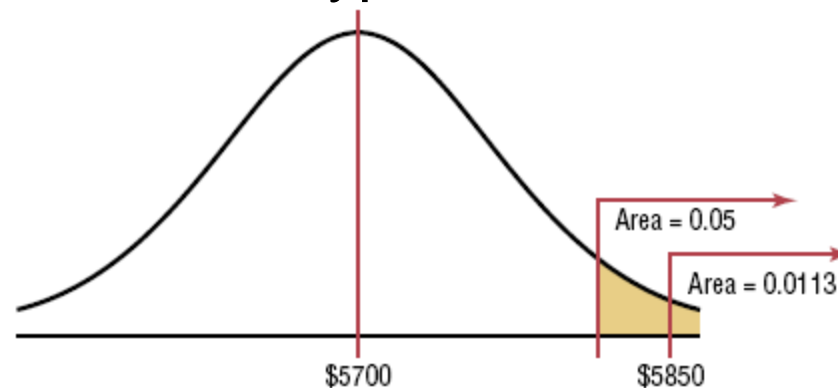
Step 5: Summarize the results.

There is enough evidence to support the claim that the tuition and fees at four-year public colleges are greater than $5700.

Note: If $\alpha = 0.01$, the null hypothesis would not be rejected.

Chapter 8
Hypothesis Testing

Section 8-2

Example 8-7

Page #420

Example 8-7: Wind Speed (option)

A researcher _claims_ that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

Step 1: State the hypotheses and identify the claim.

$H_0: \mu = 8$ (claim) and $H_1: \mu \neq 8$

Step 2: Compute the test value.

$$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{8.2 - 8}{0.6 / \sqrt{32}} = 1.89$$

Example 8-7: Wind Speed

A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the population is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

Step 3: Find the *P*-value.

The area for $z = 1.89$ is 0.9706.

Subtract: $1.0000 - 0.9706 = 0.0294$.

Since this is a two-tailed test, the area of 0.0294 must be doubled to get the *P*-value*.

The *P*-value is $2(0.0294) = 0.0588$.

Example 8-7: Wind Speed

Step 4: Make the decision.

The decision is to not reject the null hypothesis, since the *P*-value is greater than 0.05.
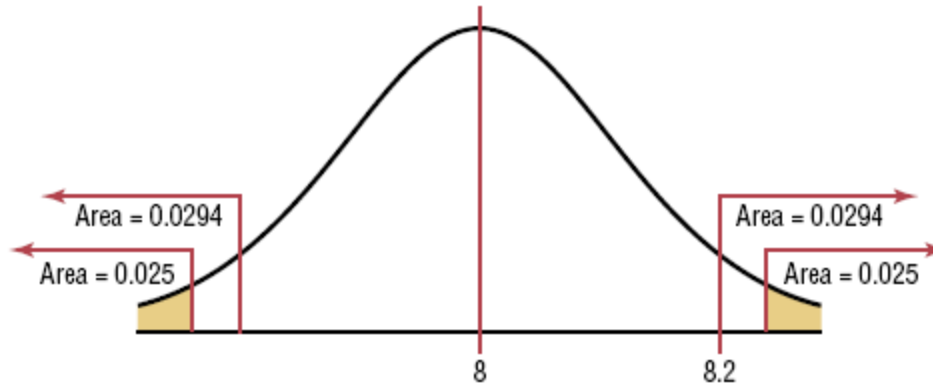
Step 5: Summarize the results.

There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.

Procedure for Finding *P*-Values
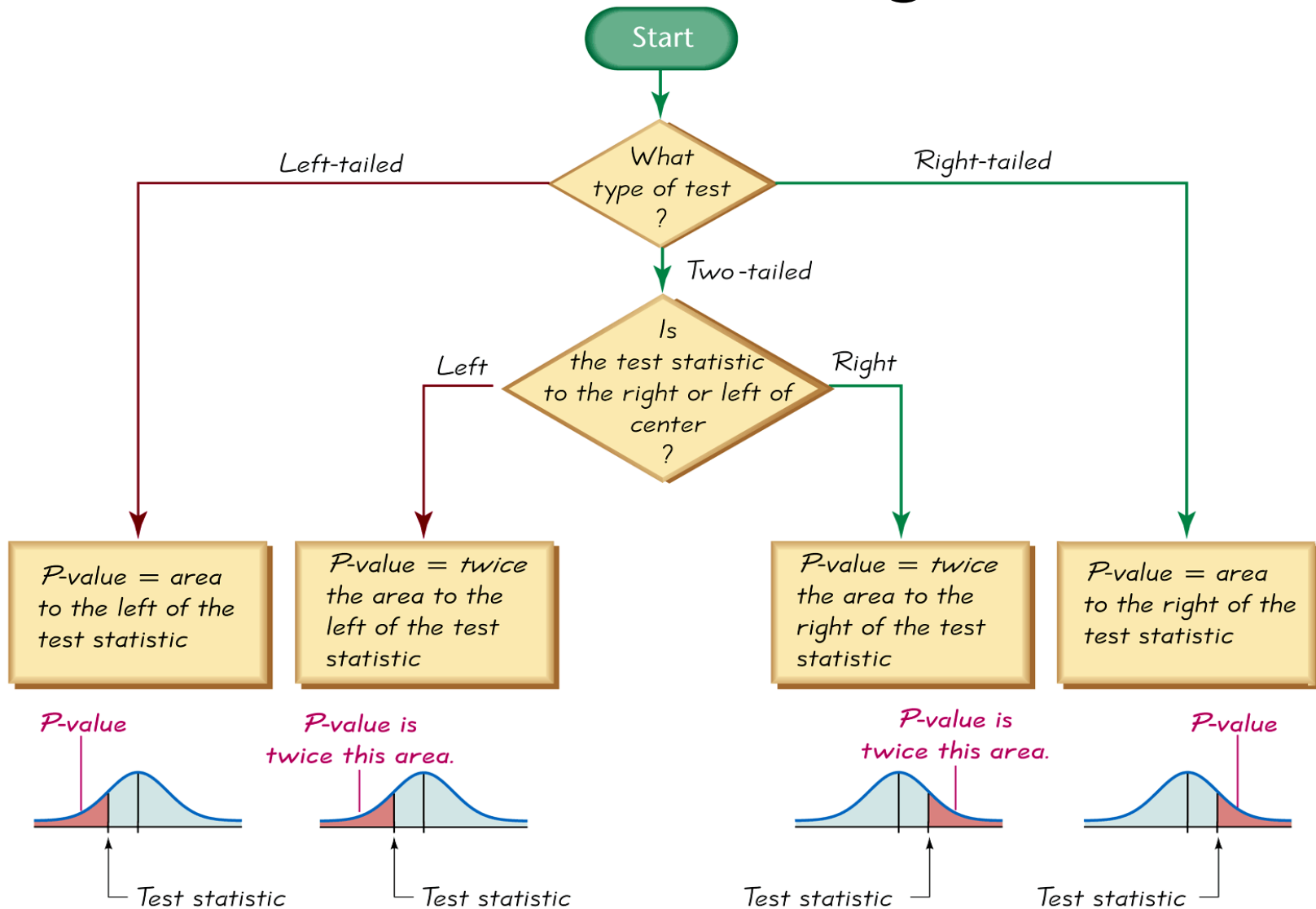
Guidelines for *P*-Values With No α

- If *P*-value $\leq$ 0.01, <u>reject the null</u> hypothesis. The difference is highly significant.

- If P-value > 0.01 but P-value $\leq$ 0.05, <u>reject the null</u> hypothesis. The difference is significant.

- If P-value > 0.05 but P-value $\leq$ 0.10, <u>consider the consequences</u> of type I error before rejecting the null hypothesis.

- If P-value > 0.10, <u>do not reject the null</u> hypothesis. The difference is not significant.

Significance

- The researcher should distinguish between **statistical significance** and **practical significance**.

- When the null hypothesis is rejected at a specific significance level, it can be concluded that the difference is probably not due to chance and thus is statistically significant. However, the results may not have any practical significance.

- It is up to the researcher to use common sense when interpreting the results of a statistical test.

8.3 *t* Test for a Mean

The <u>*t* test </u> is a statistical test for the mean of a population and is used when the population is normally or approximately normally distributed, <u>σ is unknown</u>.

The formula for the *t test is*

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

The degrees of freedom are d.f. = *n* – 1.

Note: When the degrees of freedom are above 30, some textbooks will tell you to use the nearest table value; however, in this textbook, you should round down to the nearest table value. For example, if d.f. = 59, use d.f. = 55 to find the critical value or values. This is a conservative approach.

Chapter 8
Hypothesis Testing

Section 8-3

Example 8-8
Page #428

Example 8-8: Table F

Find the critical *t* value for $\alpha = 0.05$ with d.f. = 16 for a right-tailed *t* test.

Find the 0.05 column in the top row and 16 in the left-hand column. The critical *t* value is +1.746.

d.f.	One tail, α	0.25	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.50	0.20	0.10	0.05	0.02	0.01
1							
2							
3							
4							
5							
⋮							
14							
15							
16				1.746			

Chapter 8
Hypothesis Testing

Section 8-3

Example 8-9 & 8-10

Page #428

Example 8-9: Table F

Find the critical t value for $\alpha = 0.01$ with d.f. = 22 for a left-tailed test.

Find the 0.01 column in the One tail row, and 22 in the d.f. column.

The critical value is $t = $ -2.508 since the test is a one-tailed left test.

Example 8-10: Table F

Find the critical value for $\alpha = 0.10$ with d.f. = 18 for a two-tailed t test.

Find the 0.10 column in the Two tails row, and 18 in the d.f. column.

The critical values are 1.734 and -1.734.

	Table F	The *t* Distribution					

	Confidence intervals	80%	90%	95%	98%	99%
	One tail, α	0.10	0.05	0.025	0.01	0.005
d.f.	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845

One tall   Two tails

Area α Area $\frac{\alpha}{2}$ Area $\frac{\alpha}{2}$

t $-t$ $+t$
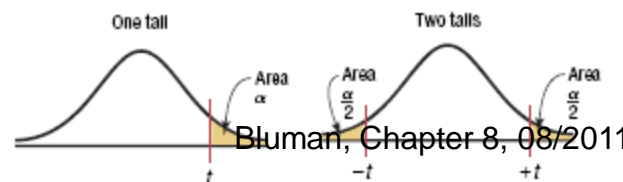
Chapter 8
Hypothesis Testing

Section 8-3

Example 8-12

Page #429

Example 8-12: Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 16.3$ (claim) and H_1: $\mu \neq 16.3$

Step 2: Find the critical value.

The critical values are 2.262 and -2.262 for $\alpha = 0.05$ and d.f. = 9.

Example 8-12: Hospital Infections

A medical investigation claims that the average number of infections per week at a hospital in southwestern Pennsylvania is 16.3. A random sample of 10 weeks had a mean number of 17.7 infections. The sample standard deviation is 1.8. Is there enough evidence to reject the investigator's claim at $\alpha = 0.05$?

Step 3: Find the test value.

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{17.7 - 16.3}{1.8/\sqrt{10}} = 2.46$$

Example 8-12: Hospital Infections

Step 4: Make the decision.

Reject the null hypothesis since 2.46 > 2.262.
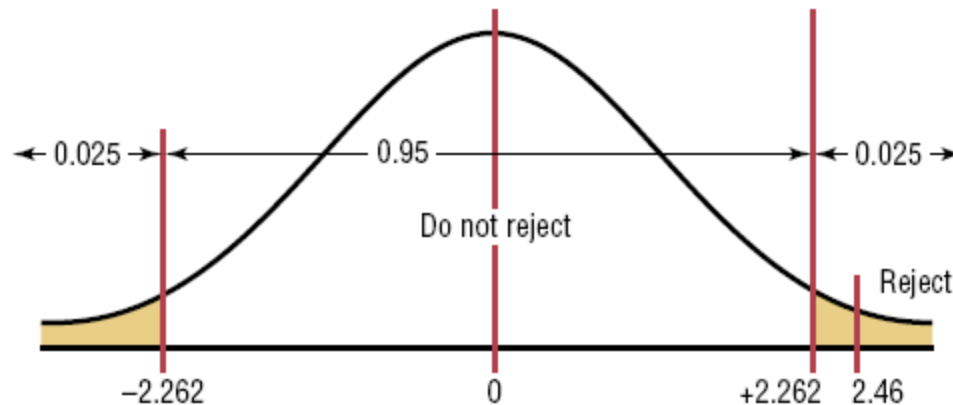
Step 5: Summarize the results.

There is enough evidence to reject the claim that the average number of infections is 16.3.

Chapter 8
Hypothesis Testing

Section 8-3

Example 8-13

Page #430

Example 8-13: Substitute Salaries

An educator claims that the average salary of substitute teachers in school districts in Allegheny County, Pennsylvania, is less than $60 per day. A random sample of eight school districts is selected, and the daily salaries (in dollars) are shown. Is there enough evidence to support the educator's claim at $\alpha = 0.10$?

60 56 60 55 70 55 60 55

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 60$ and H_1: $\mu < 60$ (claim)

Step 2: Find the critical value.

At $\alpha = 0.10$ and d.f. = 7, the critical value is -1.415.

Example 8-13: Substitute Salaries

Step 3: Find the test value.

Using the Stats feature of the TI-84, we find $\overline{X} = 58.88$ and $s = 5.08$.

L1	L2	L3	1
60			
55			
70			
55			
60			
55			

L1(9)=

```
1-Var Stats
x̄=58.875
Σx=471
Σx²=27911
Sx=5.083235752
σx=4.75493165
↓n=8
```

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{58.9 - 60}{5.1/\sqrt{8}} = -0.61 \qquad \text{-0.624}$$

Example 8-13: Substitute Salaries

Step 4: Make the decision.

Do not reject the null hypothesis since -0.624 falls in the noncritical region.
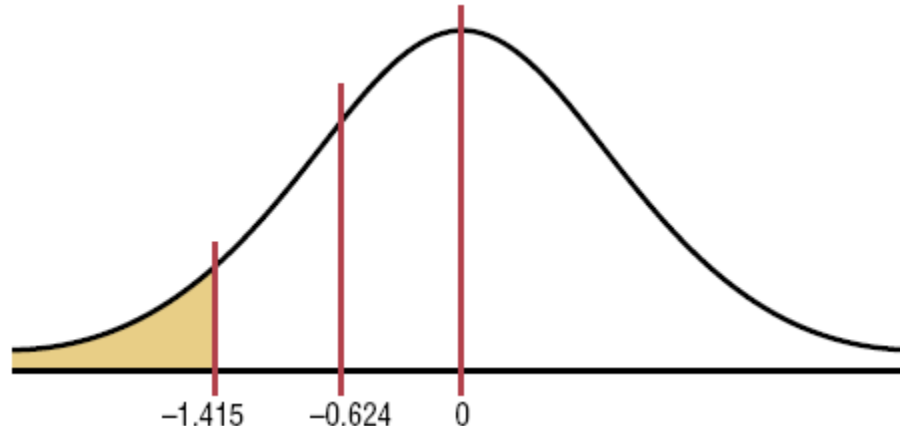
Step 5: Summarize the results.

There is not enough evidence to support the claim that the average salary of substitute teachers in Allegheny County is less than $60 per day.

Chapter 8
Hypothesis Testing

Section 8-3

Example 8-16: P-value method

Page #432

Example 8-16: Jogger's Oxygen Intake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?

Step 1: State the hypotheses and identify the claim.

H_0: $\mu = 36.7$ and H_1: $\mu > 36.7$ (claim)

Step 2: Compute the test value.

$$t = \frac{\overline{X} - \mu}{s/\sqrt{n}} = \frac{40.6 - 36.7}{6/\sqrt{15}} = 2.517$$

Example 8-16: Jogger's Oxygen Intake

A physician claims that joggers' maximal volume oxygen uptake is greater than the average of all adults. A sample of 15 joggers has a mean of 40.6 milliliters per kilogram (ml/kg) and a standard deviation of 6 ml/kg. If the average of all adults is 36.7 ml/kg, is there enough evidence to support the physician's claim at $\alpha = 0.05$?

Step 3: Find the *P*-value.

In the d.f. = 14 row, 2.517 falls between 2.145 and 2.624, corresponding to $\alpha = 0.025$ and $\alpha = 0.01$.

Thus, the *P*-value is somewhere between 0.01 and 0.025, since this is a one-tailed test.

Example 8-16: Jogger's Oxygen Intake

Step 4: Make the decision.

> The decision is to reject the null hypothesis, since the *P*-value < 0.05.

Step 5: Summarize the results.

> There is enough evidence to support the claim that the joggers' maximal volume oxygen uptake is greater than 36.7 ml/kg.

Whether to use *z* or *t*

Yes ← Is σ known? → No

Use $z_{\alpha/2}$ values and σ in the formula.*

Use $t_{\alpha/2}$ values and s in the formula.*
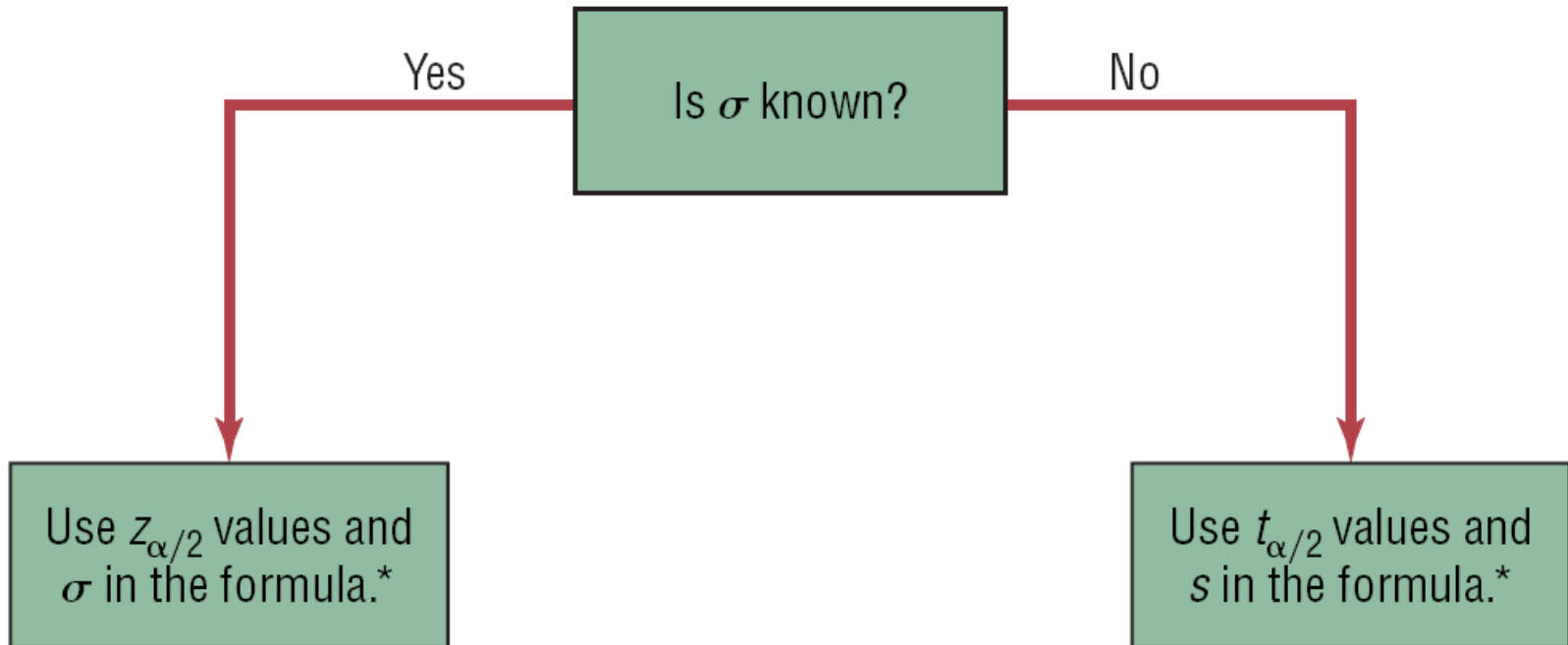
*If $n < 30$, the variable must be normally distributed.

8.4 *z* Test for a Proportion

Since a normal distribution can be used to approximate the binomial distribution when $np \geq 5$ and $nq \geq 5$, the standard normal distribution can be used to test hypotheses for proportions.

The formula for the *z* test for a proportion is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

where

$\hat{p} = \dfrac{X}{n}$ (**sample proportion**)

$p = $ **population proportion**

$n = $ **sample size**

Chapter 8
Hypothesis Testing

Section 8-4

Example 8-17

Page #438

Example 8-17: Avoiding Trans Fats

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietitian's claim?

Step 1: State the hypotheses and identify the claim.

H_0: $p = 0.60$ (claim) and H_1: $p \neq 0.60$

Step 2: Find the critical value.

Since $\alpha = 0.05$ and the test is a two-tailed test, the critical value is $z = \pm 1.96$.

Example 8-17: Avoiding Trans Fats

A dietitian claims that 60% of people are trying to avoid trans fats in their diets. She randomly selected 200 people and found that 128 people stated that they were trying to avoid trans fats in their diets. At $\alpha = 0.05$, is there enough evidence to reject the dietitian's claim?

Step 3: Compute the test value.

$$\hat{p} = \frac{\overline{X}}{n} = \frac{128}{200} = 0.64$$

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.64 - 0.60}{\sqrt{(0.60)(0.40)/200}} = 1.15$$

Example 8-17: Avoiding Trans Fats

Step 4: Make the decision.

Do not reject the null hypothesis since the test value falls outside the critical region.
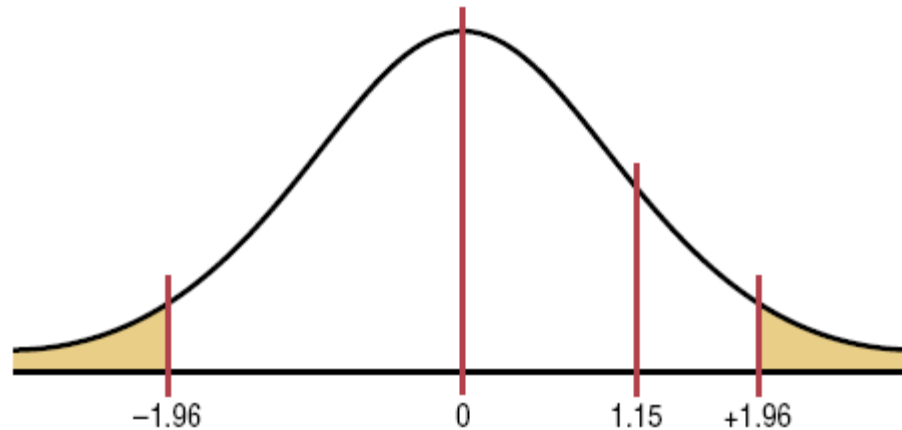
Step 5: Summarize the results.

There is not enough evidence to reject the claim that 60% of people are trying to avoid trans fats in their diets.

Chapter 8
Hypothesis Testing

Section 8-4

Example 8-18

Page #439

Example 8-18: Call-Waiting Service

A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

Step 1: State the hypotheses and identify the claim.

H_0: $p = 0.40$ (claim) and H_1: $p \neq 0.40$

Step 2: Find the critical value.

Since $\alpha = 0.01$ and the test is a two-tailed test, the critical value is $z = \pm 2.58$.

Example 8-18: Call-Waiting Service

A telephone company representative estimates that 40% of its customers have call-waiting service. To test this hypothesis, she selected a sample of 100 customers and found that 37% had call waiting. At $\alpha = 0.01$, is there enough evidence to reject the claim?

Step 3: Compute the test value.

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.37 - 0.40}{\sqrt{(0.40)(0.60)/100}} = -0.61$$

Example 8-18: Call-Waiting Service

Step 4: Make the decision.

> Do not reject the null hypothesis since the test value falls outside the critical region.
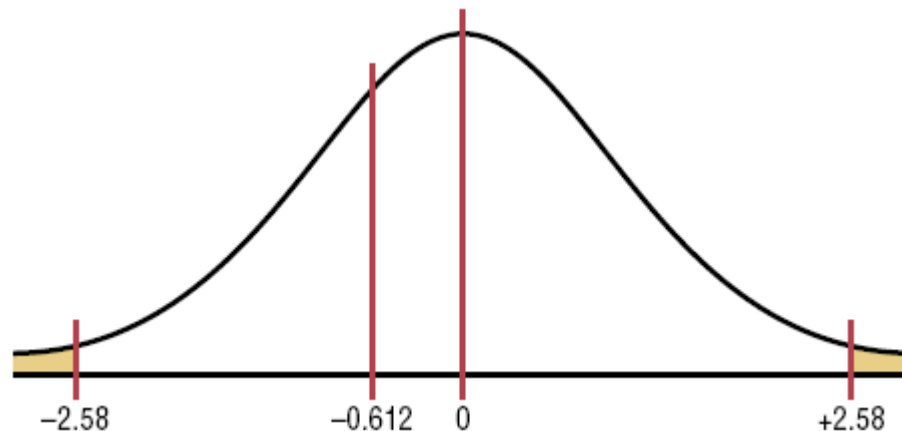
Step 5: Summarize the results.

> There is not enough evidence to reject the claim that 40% of the telephone company's customers have call waiting.

Section 8-4: Z Test for a Proportion
P-Value method

Researchers suspect that 18% of all high school students smoke at least one pack of cigarettes a day. At Wilson High School, with an enrollment of 300 students, a study found that 50 students smoked at least one pack of cigarettes a day. At $\alpha = 0.05$, test the claim that 18% of all high school students smoke at least one pack of cigarettes a day.
Use the *P* - value method.

Hind: at least one pack of … is just a label, not relevant to hypothesis test here.

$$\hat{p} = \frac{50}{300} = 0.1667 \quad p = 0.18 \quad q = 0.82$$

$X = 50$, $n = 300$, $\alpha = .05$

$$H_0 : p = 0.18 \ (\underline{\textit{claim}}) \qquad H_1 : p \neq 0.18$$

Solving Hypothesis-Testing Problems (*P*-Value Method)

Step 1	State the hypotheses and identify the claim.
Step 2	Compute the test value.
Step 3	Find the *P*-value.
Step 4	Make the decision.
Step 5	Summarize the results.

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{pq}{n}}} = \frac{0.1667 - 0.18}{\sqrt{\dfrac{(0.18)(0.82)}{300}}} = -0.60$$

Area = .2743   .2743

Area = .025   .025

$z = -.60 \qquad 0 \qquad z = .60$
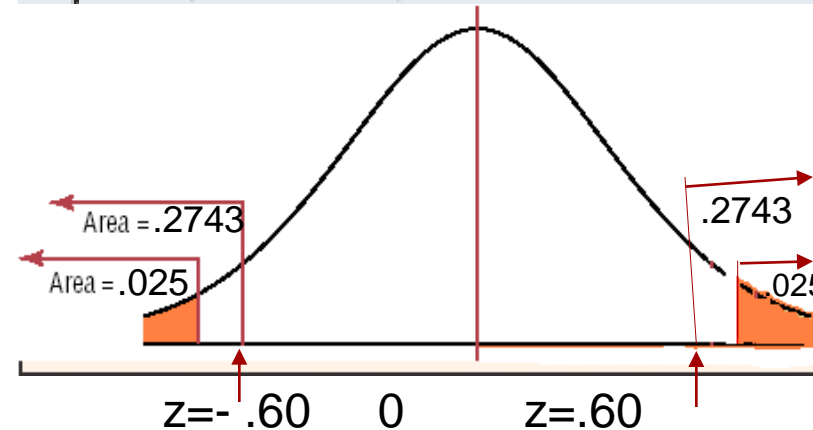
Table E: area = .2743, P-value = 2 (.2743) = .5486

Since *P* - value > 0.05, do not reject the null hypothesis.
There is not enough evidence to reject the claim that 18% of all high school students smoke at least a pack of cigarettes a day.

8.5 χ^2 Test for a Variance or a Standard Deviation

The chi-square distribution is also used to test a claim about a single variance or standard deviation.

The formula for the chi-square test for a variance is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

with degrees of freedom d.f. = $n-1$ and

n = sample size

s^2 = sample variance

σ^2 = population variance

Assumptions for the χ^2 Test for a Variance or a Standard Deviation

1. The sample must be randomly selected from the population.

2. The population must be normally distributed for the variable under study.

3. The observations must be independent of one another.

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-21

Page #445

Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when $\alpha = 0.05$ and the test is right-tailed.

Degrees of freedom					α		
	0.995	0.99	0.975	0.95	0.90	0.10	0.05
1							
2							
⋮							
15							24.996

$$\chi^2 = 24.996$$

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-22

Page #446

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.
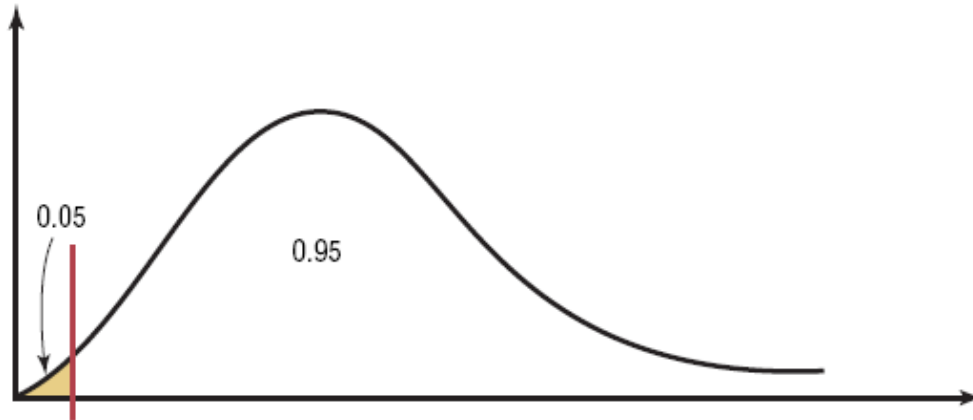
When the test is left-tailed, the α value must be subtracted from 1, that is, $1 - 0.05 = 0.95$. The left side of the table is used, because the chi-square table gives the area to the right of the critical value, and the chi-square statistic cannot be negative.

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when $\alpha = 0.05$ and the test is left-tailed.

Use Table G, looking in row 10 and column 0.95.

Degrees of freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1										
2										
⋮										
10				3.940						
⋮										

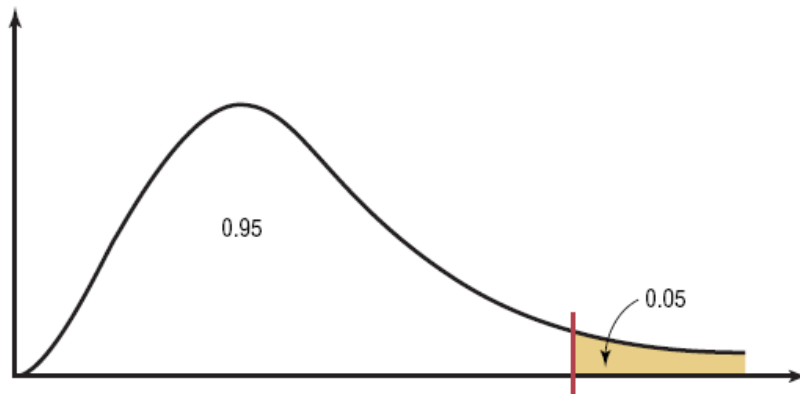$$\chi^2 = 3.940$$

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-23

Page #447

Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when $\alpha = 0.05$ and a two-tailed test is conducted.

When the test is two-tailed, the area must be split. The area to the right of the larger value is $\alpha/2$, or 0.025. The area to the right of the smaller value is $1 - \alpha/2$, or 0.975.
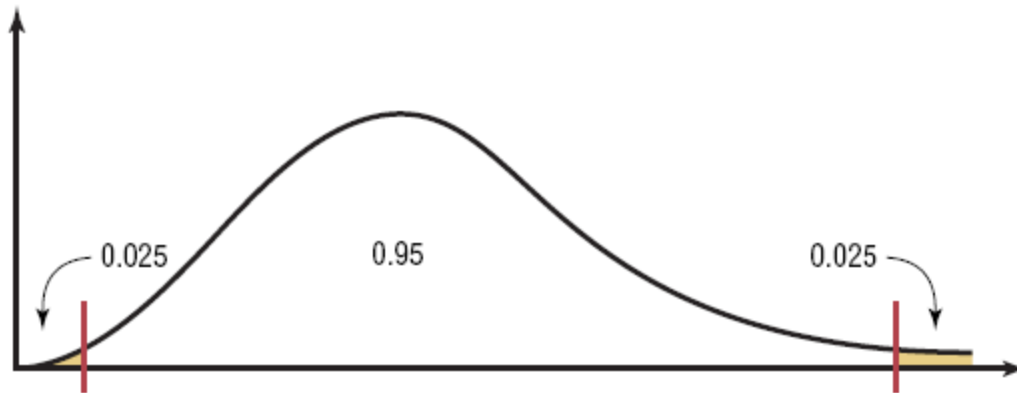
With 22 degrees of freedom, areas 0.025 and 0.975 correspond to chi-square values of 36.781 and 10.982.

Table G The Chi-Square Distribution

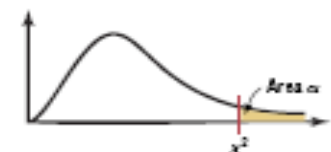| Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-24

Page #448

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is <u>less than</u> the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 225$) at $\alpha = 0.05$? Assume that the scores are normally distributed.

Step 1: State the hypotheses and identify the claim.

$$H_0: \sigma^2 = 225 \text{ and } H_1: \sigma^2 < 225 \text{ (claim)}$$

Step 2: Find the critical value. d.f. =22, α = .95, table G

The critical value is $\chi^2 = 12.338$.

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 =225$) at $\alpha = 0.05$? Assume that the scores are normally distributed.

Step 3: Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(22)(198)}{225} = 19.36$$

Example 8-24: Variation of Test Scores

Step 4: Make the decision.

Do not reject the null hypothesis since the test value 19.36 falls in the noncritical region.
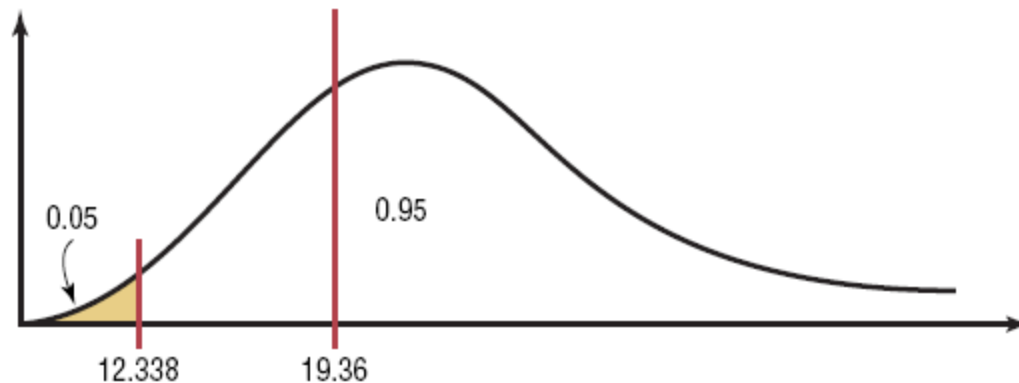
Step 5: Summarize the results.

There is not enough evidence to support the claim that the variation in test scores of the instructor's students is less than the variation in scores of the population.

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-26

Page #450

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the <u>claim</u> that the variance of the nicotine content of its cigarettes <u>is</u> 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Step 1: State the hypotheses and identify the claim.

$$H_0: \sigma^2 = 0.644 \text{ (claim) and } H_1: \sigma^2 \neq 0.644$$
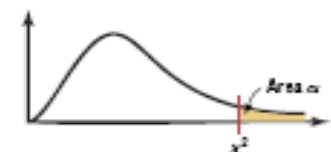
Step 2: Find the critical value.

d.f = 19, α = .05, $\alpha/2$ = .025, α_{right} = .025, α_{left} = .975

The critical values are 32.852 and 8.907.

Table G   The Chi-Square Distribution

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.262	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?
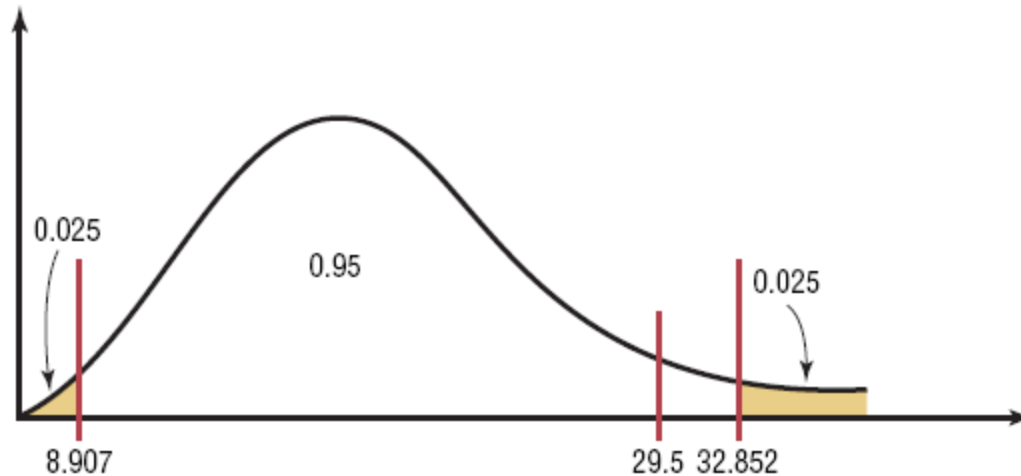
Step 3: Compute the test value.

The standard deviation s must be squared in the formula.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(19)(1.00)^2}{0.644} = 29.5$$

Example 8-26: Nicotine Content

Step 4: Make the decision.

Do not reject the null hypothesis, since the test value falls in the noncritical region.

Step 5: Summarize the results.

There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is 0.644.